A new theory of the Aharonov-Bohm effect.

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ABSTRACT. A new theory of the Aharonov-Bohm experiment, based on the calculation of the phase difference between the electronic trajectories, shows that the shifting of the interference fringes depends both on the gauge of the potential and of the location of its source with respect to the interference device. A new experiment is then suggested, in which the source of the potential is outside the electronic trajectories. The line integral of the potential along the trajectories equals zero, but the shifting ofthe fringes does not vanish.

RÉSUMÉ. Une nouvelle théorie de l'effet Aharonov-Bohm, basée sur le calcul de la différence de phase entre les trajectoires électroniques montre que l'effet dépend à la fois de la jauge du potentiel et de la position de la source par rapport au dispositif interférentiel. On propose ensuite une nouvelle expérience dans laquelle la source du potentiel est extérieure aux trajectoires. L'intégrale du potentiel le long des trajectoires est nulle, mais le déplacement des franges subsiste.

1 INTRODUCTION

The Aharonov-Bohm experiment (see : *Aharonov & Bohm, Tonomura, Olariu Iovitsu Popescu, Lochak*) was conceived in order to prove the effect of a fieldless magnetic potential on electronic interferences. The idea was to introduce, between the electronic trajectories coming from two virtual coherent sources, a magnetic string, or a thin solenoid, orthogonal to the trajectories and long enough, so that the magnetic field emanating from the extremities cannot modify the electron trajectories (Fig. 1).

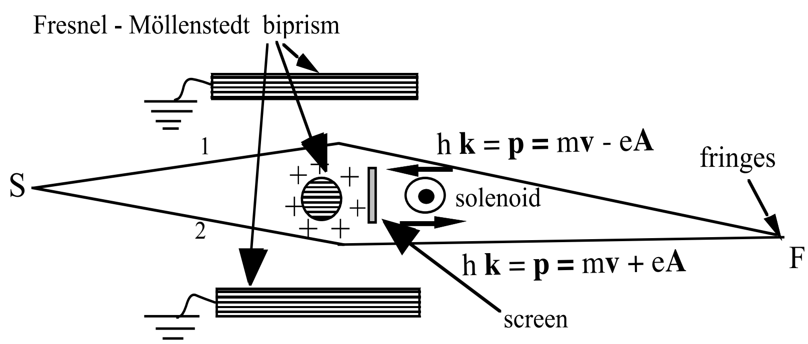

Fig. 1

Aharonov-Bohm experiment

Theoretically, in order for a magnetic flux to be trapped inside a string or a solenoid, it must be infinitely long : this is what is assumed in the calculations. But in practice, a few millimeters are sufficient because the transverse dimensions of the device are on the order of microns. As this point was contested, Tonomura (see : *Tonomura*) succeeded in substituting for the rectilinear string a microscopic toroidal magnet ($\varnothing \approx 10\ \mu m$), one electron beam passing through the hole of the torus and the other passing outside, so that the magnetic lines may be regarded as beeing entirely enclosed in the magnet.

Nevertheless, in what follows, we shall restrict ourselves to an infinite magnetic string, which is sufficient for our present object, because the subtleties of the Tonomura tori were invented in order to answer other arguments than those we are aiming at refuting in the present paper.

Let us give at first an intuitive interpretation of the Aharonov-Bohm experiment. Recall that the wave vector of an electron in a magnetic potential - even fieldless - is given by the de Broglie formula (*Broglie*) :

$$\frac{h}{\lambda}\mathbf{n} = h\mathbf{k} = \mathbf{p} = m\mathbf{v} + e\mathbf{A} \qquad (1)$$

($\mathbf{p}$ is the Lagrange momentum). This formula is a direct consequence of the identification of the principles of Fermat and of least action : it is one of the most reliable results of quantum mechanics. Therefore, it is *a priori* obvious that interference and diffraction phenomena will be influenced by the presence of a magnetic potential, independently of the presence or not of a magnetic field.

These phenomena follow from a simple change of wavelength and thus a change of phase, as may be done in optics by introducing a plate of glass into a Michelson interferometer. Besides it will be shown a little later that the phenomenon is manifestly **gauge dependent** : if we add something to $\mathbf{A}$, whether it be a gradient or not, λ is modified. Of course, it is true even when $\mathbf{A} = 0$, i.e. for the formula $\lambda = \frac{h}{mv}$ in the vacuum, which is thus gauge dependent too. This fact was emphasized by de Broglie many years ago : **electron interferences are not gauge invariant.**

In the case of the Aharonov-Bohm experiment, there are additive phases on both interfering waves, and moreover they are in opposite directions, which doubles the shift of interference fringes. We furthermore give a new proof of all this.

This remarkable effect, which proves the influence of a fieldless magnetic potential on electron waves, is shocking for those who have been convinced for a century that electromagnetic potentials are only mathematical intermediate entities. And even more shocking is the fact that formula (1) imposes an electromagnetic gauge that can be measured experimentally.

The almost unanimous opinion that gauge invariance is an absolute law is so firmly fixed in prevailing thought that even distinguished physicists (*Born & Wolf*) are led to present a wrong formula for the wavelength – writing $\lambda = \frac{h}{mv}$ in the presence of a potential, instead of the formula (1). For the same reason, Feynman managed to relate the Aharonov-Bohm effect, not with the wavelength formula (1), but with the magnetic flux trapped in the string or in the solenoid, saving in this way the gauge invariance (*Feynman*, *Lochak*)

The aim of the present work is to prove that the shifting of fringes depends on the distance from the solenoid to the experimental device and to suggest a new experiment in which the solenoid, with its magnetic flux, is outside the quadrilateral formed by the electronic trajectories, which causes the integral of $\mathbf{A}$ to vanish and makes the argument of the magnetic flux enclosed by the trajectories ineffective.

Actually, the quadrilateral itself will be removed from the calculations, rejecting to infinity the electron source and the interference fringes, which introduces negligible errors : this approximation is usual in optics.

2 A NEW THEORY OF THE AHARONOV-BOHM EFFECT

1) The effect.

The idea of Aharonov-Bohm effect was to modify electronic interferences owing to a fieldless magnetic potential created by a magnetic string, or a thin solenoid (which will be chosen here), orthogonal to the plan of interfering electronic trajectories. The solenoid must be in principle infinitely long, in order that the magnetic field emanating from the extremities cannot perturb the experiment, which is assumed in the calculations. In practice, a few millimeters are sufficient because the transverse dimensions of the device are on the order of microns. The problem was elegantly solved by Tonomura (see : Literature), substituting the rectilinear string by a microscopic torus ($10\mu m$), one electron beam passing through the torus and the other outside, so that the magnetic lines are trapped.

The Fig.1 gives a scheme of the plan of the experiment. The Young slits are obtained by a Fresnel – Möllenstedt biprism and the vertical solenoïd is introduced between the trajectories, which has led to the idea that the magnetic flux through the trajectories quadrilateral plays a role. I disagree with this idea from which we shall free ourselves, starting from the formula (1). Et is evident on this formula that electron interferences are **gauge dependent** : if we add something to $\mathbf{A}$, would it be a gradient or not, λ is modified. This is true even when $\mathbf{A} = 0$, i.e. for the de Broglie formula $\lambda = \frac{h}{mv}$ in the vacuum, which is gauge dependent too. This fact was many times emphasized by de Broglie who said : « **If gauge invariance would be general in quantum mechanics, the electron interferences would not exist ».**

In the case of Aharonov-Bohm experiment, there are additive phases on both interfering waves, and moreover they are in opposite directions, which doubles the shift of interference fringes. Our proof will be independant from the fact that a potential generates forces or not.

2) The magnetic potential of an infinetely thin and ininitely long solenoid.

We consider the case corresponding to the realized Aharonov-Bohm experiment : electrons diffracted on Young slits and falling on a magnetic solenoid. Now let us consider the Biot & Savart law (*Jackson, Tamm*), expressing the vector potential created by a magnetic dipole μ at a distance l :

$$\mathbf{B} = \frac{\mu \times \mathbf{l}}{l^3} \tag{2}$$

If the solenoid is along the z axis and ϕ themagnetic flux trapped in the solenoid. The potential in a point P is :

$$\mathbf{B} = \phi \int_{-\infty}^{+\infty} \frac{\mathbf{dZ} \times \mathbf{l}}{l^3} = \phi \int_{-\infty}^{+\infty} \frac{\mathbf{dZ} \times \mathbf{MP}}{|\mathbf{MP}|^3} \quad ; \qquad \begin{cases} \mathbf{MP}^2 = \mathbf{l}^2 = x^2 + y^2 + (z - Z)^2 \\ \mathbf{dZ} \times \mathbf{MP} = \{-y\,dZ, x\,dZ, 0\} \end{cases} \tag{3}$$

which gives :

$$B_x = -\phi\, y \int_{-\infty}^{+\infty} \frac{dz}{\left[x^2 + y^2 + (z-Z)^2\right]^{3/2}}; \quad B_y = \phi\, x \int_{-\infty}^{+\infty} \frac{dz}{\left[x^2 + y^2 + (z-Z)^2\right]^{3/2}}; \quad B_z = 0 \qquad (4)$$

thus : $B_x = -2\phi \dfrac{y}{x^2+y^2}$; $B_y = 2\phi \dfrac{x}{x^2+y^2}$; $B_z = 0$ (5)

3) Theory of the effect.

The commonly admitted theories are too complicated (*Olariu S., Iovitsu Popescu*). For the physical bases, one can read the brillant book of *Tonomura*. The geometrical optics approximation and the phase $\varphi = S/\hbar$ of de Broglie's wave are sufficient to find the fringes. S is the principal Hamilton function which obeys the Hamilton-Jacobi equation where $\varepsilon = 2\phi$ is twice the magnetic flux in the solenoid :

$$2m\frac{\partial S}{\partial t} = \left(\frac{\partial S}{\partial x} + \varepsilon \frac{y}{x^2+y^2}\right)^2 + \left(\frac{\partial S}{\partial y} - \varepsilon \frac{x}{x^2+y^2}\right)^2 \qquad (6)$$

The wave propagates from $x = -\infty$ to $x = +\infty$. The slits are parallel to Oy, at a distance $\pm\dfrac{a}{2}$ from C :

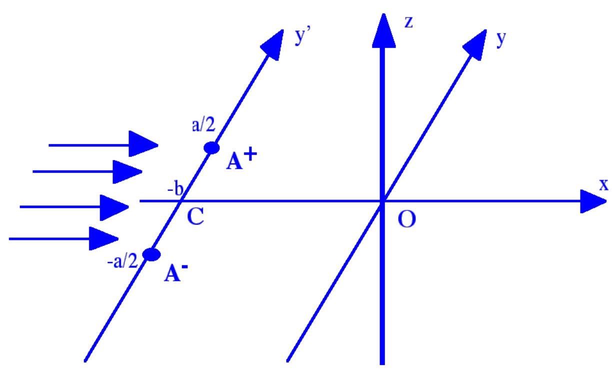

Fig. 2
Aharonov-Bohm scheme

The potential appearing in (6) is a gradient because :

$$-\frac{y}{x^2+y^2} = \partial_x Arctg\frac{y}{x} \; ; \; \frac{x}{x^2+y^2} = \partial_y Arctg\frac{y}{x} \qquad (7)$$

So, the equation (6) may be written as :

$$2m\frac{\partial S}{\partial t} = \left(\frac{\partial S}{\partial x} + \varepsilon B_x\right)^2 + \left(\frac{\partial S}{\partial y} + \varepsilon B_y\right)^2 \qquad (8)$$

with the potential :

$$\mathbf{B} = \left\{B_x, B_y, 0\right\} = \frac{1}{k_0}\nabla I_2; \quad I_2 = Arctg\frac{y}{x} \tag{9}$$

It is easy to verify that the potential B and the invariant I_2 so defined satisfy the equations (NM) of the spin 0 photon (*Lochak G.* 3, eq. 22) with $W = 0$ and B independant of t.

The equation (6) is immediately integrated, defining the phase :

$$\Sigma = S - \varepsilon Arctg\frac{y}{x} \tag{10}$$

which gives :

$$2m\frac{\partial\Sigma}{\partial t} = \left(\frac{\partial\Sigma}{\partial x}\right)^2 + \left(\frac{\partial\Sigma}{\partial y}\right)^2 \tag{11}$$

Chosing a complete integral of (12) : and then of (6), owing to (8) :

$$\Sigma = Et - \sqrt{2mE}\left(x\cos\theta_o + y\sin\theta_o\right) \tag{12}$$

we have an integral of (6) or (8) :

$$S = Et - \sqrt{2mE}\left(x\cos\theta_o + y\sin\theta_o\right) + \varepsilon\, Arctg\frac{y}{x} \tag{13}$$

or, in polar coordinates $x = r\cos\theta$, $y = r\sin\theta$:

$$S = Et - \sqrt{2mE}\, r\cos\left(\theta - \theta_o\right) + \varepsilon\,\theta \tag{14}$$

The Jacobi theorem gives the trajectories (**wave rays**) :

$$\frac{\partial S}{\partial\theta_o} = \sqrt{2mE}\left(x\sin\theta_o - y\cos\theta_o\right) = \mu \;; \qquad \frac{\partial S}{\partial E} = t - \sqrt{\frac{m}{2E}}\left(x\cos\theta_o + y\sin\theta_o\right) = t_o \tag{15}$$

and finally[1], with $E = \frac{1}{2}m\mathrm{v}^2$, we have the motion along the rays :

$$x\cos\theta_o + y\sin\theta_o = \mathrm{v}\left(t - t_o\right) \tag{16}$$

[1] We are obviously far from relativity.

We see that the *rays* (electron trajectories) defined in (15) are **orthogonal to the moving planes but they are not orthogonal to the equal phase surfaces** (13) - (14), except far from the magnetic string $(x \to \infty)$, when the potential term of the order ε becomes negligible.

Therefore, despite the presence of the potential, the electronic trajectories remain rectilinear and are not deviated because there is no magnetic field. The velocity $\mathrm{v} = Const$ **remains the one of the incident electrons because of the conservation of energy.**

But the diffraction of waves through the slits $\mathbf{A}^+$ and $\mathbf{A}^-$ creates, for the electron trajectories, an interval of possible angles θ_o, among which are the angles of the interference fringes, modified by the magnetic potential :

There is no deviation of the electrons, only a deviation of the angles of phase synchronization between the waves issued from $\mathbf{A}^+$ and $\mathbf{A}^-$. This is the Aharonov-Bohm effect, which is in accordance with the definition of the spin 0 photon (*Lochak G.* 3, eq. 22**).**

4) The shifting of interference fringes.

To find the shift of interference fringes, we must look at the orthogonal lines to the equal phase surfaces S. They are enveloped by the Lagrange momenta i.e. by the de Broglie wave-vectors, in accordance with the formula (1), while the rays (13) are the impulse lines m**v**.

The momenta are :

$$p_x = -\frac{\partial S}{\partial x} = \sqrt{2mE}\cos\theta_o - \varepsilon\frac{y}{x^2+y^2}$$
$$p_y = -\frac{\partial S}{\partial y} = \sqrt{2mE}\sin\theta_o + \varepsilon\frac{x}{x^2+y^2} \qquad (17)$$

Hence the equation :

$$\frac{dx}{\sqrt{2mE}\cos\theta_o - \varepsilon\frac{y}{x^2+y^2}} = \frac{dy}{\sqrt{2mE}\sin\theta_o + \varepsilon\frac{x}{x^2+y^2}} . \qquad (18)$$

Thanks to the integral combinations $xdx + ydy$ and $xdy - ydx$, we find in polar coordinates :

$$r\sin(\theta - \theta_o) - \Lambda\log\frac{r}{\Lambda} = c \quad (= Const), \qquad \Lambda = \frac{\varepsilon}{\sqrt{2mE}} \qquad (19)$$

or in Cartesian coordinates :

$$y\cos\theta_o - x\sin\theta_o - \Lambda\log\frac{\sqrt{x^2+y^2}}{\Lambda} = c \qquad (20)$$

Comparing with (15), we see that the lines orthogonal to the phase planes become parallel to the rays, far from the magnetic string. It is worth noting that these lines and the phase velocity $V = \frac{h\nu}{p}$ (which we cannot calculate here because the phase frequency is correct only in relativity) depends on the potential through the momentum **p,** contrary to the electron trajectories (16) and velocities.

In other words, the electrons (i.e. energy) do not follow the phase propagation, neither in velocity, nor in trajectory. The same happens in crystal optics : the phase propagation depends on the **inductions** (that is on the polarization of the medium), while the propagation of energy is given by the Poynting vector, which is only defined by **fields** and does not depend on the polarization (*Born & Wolf*).

Let us consider a plane wave propagating along $Ox\ (\theta_o = 0)$.The holes $\mathbf{A}^+$ and $\mathbf{A}^-$ will emit in the half space $x > 0$ two waves S^+ and S^-. According to (13), we have :

$$S^+ = Et - \sqrt{2mE}\left[x + b + \left(y - \frac{a}{2}\right)\theta_o\right] + \varepsilon\, Arctg\frac{y}{x}$$
$$S^- = Et - \sqrt{2mE}\left[x + b + \left(y + \frac{a}{2}\right)\theta_o\right] + \varepsilon\, Arctg\frac{y}{x} \qquad (21)$$

We have taken into account the smallness of θ_o : $\cos\theta_o \approx 1$, $\sin\theta_o \approx \theta_o$. Let us now suppose that $t = 0$ when $x = -b$, introducing :

$$\xi = Arctg\frac{a}{2b} \qquad (22)$$

The initial waves S+ and S-, in $\mathbf{A}^+$ and $\mathbf{A}^-$ $\left(x = -b, y = \pm\frac{a}{2}\right)$, are :

$$S_o^+ = -\varepsilon\xi\,, S_o^+ = +\varepsilon\xi \qquad (23)$$

Now, let us note that, in all the known experiments, the magnetic string, or the solenoid, was very close to $\mathbf{A}^+$and $\mathbf{A}^-$. The authors say : « in the shadow » of the electrostatic fiber of the Fresnel-Möllenstedt biprism (*Olariu, Iovitsu Popescu*), as it is shown on the Fig. 1. Therefore, in $\mathbf{A}^+$and $\mathbf{A}^-$, the distance b is very small and :

$$\xi \propto \frac{\pi}{2}, \text{ so that : } S_o^- = +\varepsilon\,\frac{\pi}{2},\ S_o^+ = -\varepsilon\,\frac{\pi}{2} \qquad (24)$$

Therefore, we see that in $\mathbf{A}^+$and $\mathbf{A}^-$, at the beginning of the trajectories, the phases depend on the potential only through the value of ε. On the contrary, at the other end of the trajectories, on the interference fringes, far from $\mathbf{A}^+$ and $\mathbf{A}^-$, the distance is of the order of $15\,cm$, while $a,b \propto 10^{-4}cm$, which justifies the approximation of parallel trajectories for the waves S^+ and S^- in the vicinity of the fringes.

Close to the fringes, the term $\varepsilon\theta = \varepsilon\, Arctg(y/x)$ has practically the same value for S^+ and S^- so that the difference $\varepsilon\delta\theta$ would be of the third order and it disappears. In other words, contrary to the origin, the potential has no more influence on the fringes. Finally, the interference field is defined by :

$$S^+ - S_o^+ = Et - \sqrt{2mE}\left[x + b + \left(y - \frac{a}{2}\right)\theta_o\right] + \varepsilon\xi$$
$$S^- - S_o^- = Et - \sqrt{2mE}\left[x + b + \left(y + \frac{a}{2}\right)\theta_o\right] - \varepsilon\xi \qquad (25)$$

Introducing the wavelengh $\lambda = \frac{h}{\sqrt{2mE}}$, the phase difference between the two waves will be :

$$\Delta\varphi = \frac{\Delta S}{h} = \frac{a\theta_o}{\lambda} + \frac{2\varepsilon\xi}{h} \qquad (26)$$

The first term gives the standard Young fringes, **the second term is the Aharonov-Bohm effect**. ξ is half the angle under which the Young slits are seen from the solenoid so that ξ entails a dependence of the effect on the position of the string, which is in principle experimentally testable : the effect must decrease when the distance b increases.

3 A NEW EXPERIMENT

We shall now suggest an experiment inspired by that of Aharonov-Bohm, but which is such that **the circular integral along the electron trajectories equals zero an thus cannot have any relation with the fringe shift**. This experiment was already suggested in (*Feynman*) but as an intuitive argument. Here we give the exact calculation.

The idea is to substitute the magnetic string included between the electronic trajectories by *two strings on both sides* (Fig. 3 and 4). In principle, one string would be enough, but we shall see that the effect is smaller than Aharonov-Bohm's, so that it is useful to double it. Owing to the new position of strings, the magnetic flux through the closed line of trajectories will be equal to zero because the potential is still a gradient and its source is outside. The effect remains, but the problem of gauge invariance is clearly irrelevant.

The Hamilton-Jacobi equation becomes :

$$2m\frac{\partial S}{\partial t} = \left[\frac{\partial S}{\partial x} + \varepsilon\left(\frac{y-c}{x^2 + (y-c)^2} + \frac{y+c}{x^2 + (y+c)^2}\right)\right] +$$
$$+\left[\frac{\partial S}{\partial y} - \varepsilon\left(\frac{x}{x^2 + (y-c)^2} + \frac{x}{x^2 + (y+c)^2}\right)\right] \qquad (16)$$

We see that, according to Fig. 3 and 4, the magnetic strings are parallel to Oz, and cut the plane xOz in two points at a distance c from Oz. We suppose :

$$c > \frac{a}{2} \qquad (17)$$

in order to put the strings outside the trajectories.

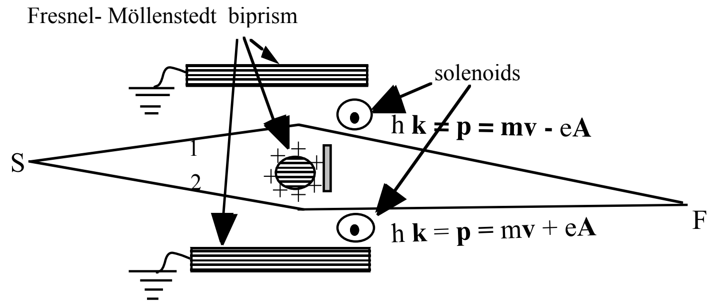

Fig. 3

New experiment

Paralleling the relations (7), we have now:

$$-\left(\frac{y-c}{x^2+(y-c)^2}+\frac{y+c}{x^2+(y+c)^2}\right)=\frac{\partial}{\partial x}Arctg\frac{y-c}{x}+\frac{\partial}{\partial x}Arctg\frac{y+c}{x}$$
$$\left(\frac{x}{x^2+(y-c)^2}+\frac{x}{x^2+(y+c)^2}\right)=\frac{\partial}{\partial y}Arctg\frac{y-c}{x}+\frac{\partial}{\partial y}Arctg\frac{y+c}{x} \qquad (18)$$

And in analogy with (10) :

$$\Sigma = S-\varepsilon\left(Arctg\frac{y-c}{x}+Arctg\frac{y+c}{x}\right) \qquad (19)$$

Introducing (19) in (16), we get the equation (11) again, with the complete integral (7), and finally a complete integral of (16), analogous to (13) :

$$S = Et-\sqrt{2mE}\left(x\cos\theta_o+y\sin\theta_o\right)+$$
$$+\varepsilon\left(Arctg\frac{y-c}{x}+Arctg\frac{y+c}{x}\right) \qquad (20)$$

We shall not repeat the whole preceding theory. The most important thing is to note that the electron trajectories are the same straight lines as before, for the same reason : the absence of magnetic field. We find equations (12), (13), (14) again, for the wave rays. The Lagrange momenta (de Broglie wave vectors, up to a factor h) are now :

$$p_x=-\frac{\partial S}{\partial x}=\sqrt{2mE}\cos\theta_o-\varepsilon\left(\frac{y-c}{x^2+(y-c)^2}+\frac{y+c}{x^2+(y+c)^2}\right)$$
$$p_y=-\frac{\partial S}{\partial y}=\sqrt{2mE}\sin\theta_o+\varepsilon\left(\frac{x}{x^2+(y-c)^2}+\frac{x}{x^2+(y+c)^2}\right) \qquad (21)$$

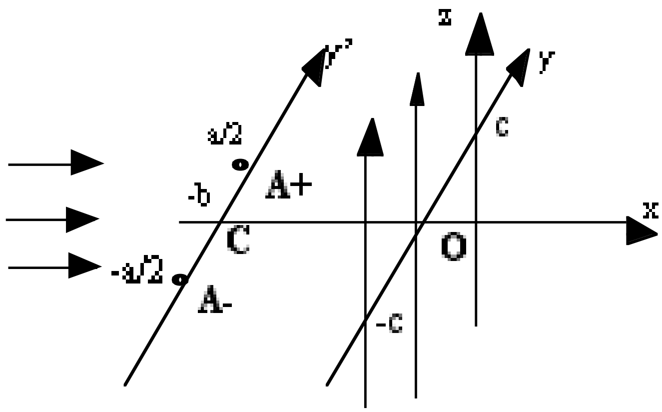

Fig 4
New experiment scheme

The equations of the orthogonal lines of phase would be useless for the prediction of the physical effect : but it is interesting to perform the integration only once, on the example (16), in order to show the difference between rays and phase lines.

The shifting of interference fringes.

Let us look once more at a plane wave coming from $x = -\infty$ to the plane $x = -b$, and diffracting through the holes $\mathbf{A}^+$ and $\mathbf{A}^-$. The angle θ_o is small again and we have, owing to (28) and in analogy with (19), two waves :

$$S^{\pm} = Et - \sqrt{2mE}\left[x + b + \left(y \mp \frac{a}{2}\right)\theta_o\right] + \\ + \varepsilon\left(Arctg\,\frac{y-c}{x} + Arctg\,\frac{y+c}{x}\right) \qquad (30)$$

For we have, up to a common constant additive term :

$$S_o^+ = \varepsilon(\eta - \zeta)\,,\, S_o^- = -\varepsilon(\eta - \zeta) \qquad (31)$$

with the definitions :

$$\eta = Arctg\frac{c - \frac{a}{2}}{b}\,;\; \zeta = Arctg\frac{c + \frac{a}{2}}{b} \qquad (32)$$

Disregarding, as in (19), the small terms corresponding to the potential near the interference field (great values of x), we find the analogue of (22) for the phase differences for the waves coming from $\mathbf{A}^+$ and $\mathbf{A}^-$:

$$S^+ - S_o^+ = Et - \sqrt{2mE}\left[x + b + \left(y - \frac{a}{2}\right)\theta_o\right] - \varepsilon(\eta - \zeta)$$
$$S^- - S_o^- = Et - \sqrt{2mE}\left[x + b + \left(y + \frac{a}{2}\right)\theta_o\right] + \varepsilon(\eta - \zeta) \tag{33}$$

Introducing the wavelength $\lambda = \frac{h}{\sqrt{2mE}}$, we can deduce the phase difference between the two waves, just as in (23) :

$$\Delta\varphi = \frac{\Delta S}{h} = \frac{a\theta_o}{\lambda} + \frac{2\varepsilon}{h}(\zeta - \eta) \tag{34}$$

We find again a first term, corresponding to the Young interferences, and *a second one analogous to the Aharonov-Bohm effect*. This term is smaller, for the obvious reason that each magnetic string produces a shift on the nearest trajectory, but unfortunately it also produces a shift on the other one, and this second shift is in the same direction as the first one because both trajectories are on the same side of the string, whereas they were on opposite sides in the case of Aharonov-Bohm, so that the phase shifts on the trajectories were opposite too. This is why we find now, instead of a factor ξ, the difference $(\zeta - \eta)$, with $\eta, \zeta > 0$ because we have chosen $c > \frac{a}{2}$ in order for the string to be outside the trajectories. Nevertheless, the first shift dominates because the second trajectory is farther from the string than the first one, so that the effect does exist. And since we have two strings, the effect is doubled : hence the factor two before ε in (34).

Let us take, as an example, $c = a$. Then we have :

$$\eta = Arctg\frac{a}{2b}, \zeta = Arctg\frac{3a}{2b} \Rightarrow \max(\zeta - \eta) = 0,52 \tag{35}$$

The maximum value of $(\zeta - \eta)$ is obtained for $b = a\frac{\sqrt{3}}{2}$. Comparing the maximum value of the Aharonov-Bohm shift in (20) : $\xi \propto \frac{\pi}{2} \approx 1,57$ with the maximum shift in (34) : $(\zeta - \eta) \approx 0,52$, we see that the effect predicted here is three times smaller. But this is not important because the aim was not to give another proof of the interference shift due to a fieldless potential (the Aharonov-Bohm proof is excellent), but to prove that an effect of the same type can be obtained with an experiment which cannot be explained in terms of a line integral which here obviously vanishes.

4 THE QUESTION OF GAUGE INVARIANCE.

There is only one problem in an interference phenomenon : **where are the fringes ?** And the answer is given by the phase difference between two waves coming from two coherent sources.

Curiously, the calculation of this phase difference is at the basis of all the interference phenomena except the Aharonov-Bohm effect ! **The interference is taken for granted and the only question is to find the shift without damaging the gauge invariance**. This is why the circular integral of **A** plays the central role. But circular integral cannot give the interfringe.

Therefore, the phenomenon is calculated in two parts : a) The « free » interference without potential. b) The shift due to the potential, considered separately and which is absent from the calculation of the phase differences, thus forgetting the geometry of the experiment. It is for this reason that the location of the solenoid and the form under which the potential enters the expression of the phase are forgotten.

If there is something new in the present paper, it is precisely an attempt to come back to the old problems and methods of interference phenomena owing to a simple calculation of phases.

Now we shall go back to the phases given by formulae (30), which include the case of (19), adding in an arbitrary gauge term $f(x,y)$. We find :

$$
\begin{aligned}
S^{+} &= Et - \sqrt{2mE}\left[x + b + \left(y - \frac{a}{2}\right)\theta_o\right] \\
&+\varepsilon\left(Arctg\frac{y-c}{x} + Arctg\frac{y+c}{x} + f(x, y+c)\right) \\
S^{-} &= Et - \sqrt{2mE}\left[x + b + \left(y + \frac{a}{2}\right)\theta_o\right] \\
&+\varepsilon\left(Arctg\frac{y-c}{x} + Arctg\frac{y+c}{x} + f(x, y-c)\right)
\end{aligned}
\qquad (36)
$$

Close to the slits, we have, generalizing (31) :

$$
\begin{aligned}
S_o^{+} &= \varepsilon\left[\eta - \zeta + f\left(\frac{a}{2} - c\right) + f\left(\frac{a}{2} + c\right)\right] \\
S_o^{-} &= \varepsilon\left[-(\eta - \zeta) + f\left\{-\left(\frac{a}{2} + c\right)\right\} + f\left\{-\left(\frac{a}{2} - c\right)\right\}\right]
\end{aligned}
\qquad (37)
$$

and the phase difference (34) becomes :

$$
\Delta\varphi = \frac{a\theta_o}{\lambda} + \frac{2\varepsilon}{h}(\eta - \zeta) + \frac{2\varepsilon}{h}\begin{bmatrix} f\left(\frac{a}{2} - c\right) + f\left(\frac{a}{2} + c\right) \\ -f\left\{-\left(\frac{a}{2} + c\right)\right\} - f\left\{-\left(\frac{a}{2} - c\right)\right\}\end{bmatrix} \qquad (38)
$$

Clearly, except if $f(x,y)$ is even in y, **the phase difference is modified and the phenomenon is not gauge invariant.**

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