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A new electromagnetism with 4 fundamental photons : Electric and magnetic, with spin 1 and spin 0

PART 1 : THEORY OF LIGHT

§1) Theory of light and wave mechanics, an historical recall.

This chapter is an introduction to a new theory of light and gravitation (the last at a linear approximation) which generalizes, owing to the idea of magnetic monopole, the de Broglie theory of light and gravitation based on his theory of spin particles. The idea of leptonic monopole - and its consequences - is the new concept added to the Broglie theory. On the contrary, other ideas that appear in the new theory including the « magnetic photon » were implicitely present in a hidden form, in the de Broglie theory of spin particles ; but curiousely they remain neither exploited nor even noticed until recent years. This is the reason of the following short historical recall.

The de Broglie theory of spin particles started from his works on the theory of light which began as a dynamical theory of the Einstein photon. In that time (1922) the wave mechanics did not yet exist : it appeared a little later, precisely from this dynamical theory of Einstein's « light quanta » (*Broglie* 1).

De Broglie tried at first, a kind of test of the photon hypothesis, going as far as possible in the radiation theory, in a purely *corpuscular* way, in the spirit of Newton, but introducing relativistic mechanics, kinetic theory and thermodynamics ; nevertheless, without electromagnetism because de Broglie *aimed to find where and in what form the waves become necessary.*

He considered the Einstein « light quanta », not yet called « photons », as true particles (he said : "atoms of light") with a *small proper mass,* obeing the laws of relativistic mechanics. Starting from a purely corpuscular point of view he got several results previously considered as consequences of electromagnetism :

- For instance if $E = mc^2 = 1/\sqrt{1-\mathrm{v}^2/c^2} = \textit{total energy}$, the relativistic form of the momentum G is : $G = mc = E/c$, from which de Broglie obtained the correct relation $p = \rho/3$ between pressure and energy density of the black radiation (*Broglie* 1), first proved by Boltzmann, and later ascribed to Maxwell's theory[1].
- Applying relativity, de Broglie gave the correct mean energy $3\,kT$ for the photon, instead of the half value $3/2kT$ of classical theory of gas. This energy was usually considered as the sum of electric and magnetic energies, while it is a simple consequence of relativistic kinematics.
- At last, de Broglie obtained the formula of the Doppler effect, from the relativistic addition of velocities and Planck's law of quanta.

After these first results, de Broglie realized that, all which he was saying was not at all restricted to light and photons, but could be said about every particle. So, he attached a frequency to each material particle by the equality : $mc^2 = h\nu$. This equality brought him, if not yet to the wave at least to a

[1] It is curious to note that Planck found twice this result, due to the omission of relativity, (absolutely astonishing from Max Planck !). So, he wrote : $E = 1/2\, m\mathrm{v}^2 \Rightarrow G = m\mathrm{v}{=}2W/\mathrm{v} \Rightarrow \mathrm{p}{=}2\rho/3$, with an erroneous factor 2, considered by the opponents to Einstein as an argument against the photon hypothesis (*Broglie* 1) !…

frequency that he prescribed to an "internal clock» of the particle, which was not far from the Newton conceptions. But he rapidly understood that such an interpretation is not relativistically invariant because, if ν is an internal frequency of a particle, it is submitted to the slowing down of clocks while m will increase with the velocity. The de Broglie « illuminating idea » (according to his proper words) was that, on the contrary, the frequency of a wave would have the same variance as m so that the equality $mc^2 = h\nu$ becomes relativistically invariant and defines univocally ν from m. **It was the starting point of wave mechanics**.

It must be stressed that de Broglie considered from the very beginning that the photon had a mass : a mass far smaller than the one of an electron, but a « true » mass that includes the photon in a description of all the particles of the universe. Nevertheless, such a theory of light could not be developed with the Schrödinger or Klein-Gordon equations, because the first is non relativistic and the waves of both equations are not polarized.

The situation became different with the Dirac equation for which de Broglie was immediately enthusiastic, because he saw in it a possible beginning for a theory of light (*Broglie 2,*3,4) : the equation was relativistic, with a four component wave function (therefore a polarization), a spin : the axial vector that he had predicted for light[2], and a second rank tensor : $M_{\mu\nu} = \overline{\psi}\gamma_{\mu}\gamma_{\nu}\psi$, antisymmetric as the electromagnetic tensor, despite it was not a wave.

Nevertheless, the elements of the Dirac equation, could not be directly applied to a photon : the wave has neither the variance of a vector, nor of an antisymmetric tensor (such a tensor $\left(\bar{\psi}\gamma_{\mu}\gamma_{\nu}\psi\right)$ is present in the theory, but it is not the wave) ; the spin rotates twice too slowy and the particle is a fermion, not a boson as it was already welllnown. Nevertheless, the way was not obstructed as precedingly ; because the different elements did exist though in a distorded form.

After some first attempts (*Broglie 2,3*), **de Broglie realized that a photon cannot be an elementary particle, but the fusion of a pair :** perhaps of a spin 1/2 corpuscle and its « anticorpuscle » (this word appeared here for the first time) both obeing a Dirac equation (*Broglie 3*).

The creation and annihilation of pairs suggested that a photon could result from the "fusion" of an electron-positron pair linked by an electrostatic force. The smallness of the photon mass could be a consequence of a relativistic mass-defect. But the introduction of an electrostatic force is a boot-strap because a theory of photon is a theory of electromagnetism, so that the electrostatic force must be a *consequence* of the theory, not an a priori hypothesis.

So, de Broglie recognizing that the choice of conjugated particles was impossible, he supposed that the photon is a pair of neutrino-antineutrino or, more generally, the center of mass of **a couple of Dirac particles.** He published the equation in 1934 (*Broglie 3, 4*) and he developed the theory during many years.

§2) De Broglie's method of fusion *(Broglie, 3,4,7,8)*.

Let us take at first, as an example, a pair of identical, ordinary particles of mass m, obeying the Schrödinger equation, with respective coordinates $\left(x_1, y_1, z_1\right)$ and $\left(x_2, y_2, z_2\right)$. Their center of mass is :

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[2] In his 1922 paper (Broglie 1) de Broglie wrote: " *A more complete theory of quanta of light must introduce a polarization in such a way that : to each atom of light would be linked an internal state of right or left polarization represented by an axial vector with the same direction as the propagation velocity*." It was the idea of spin, because it was shown later, that when the velocity of a particle tends to the velocity of light, the space components of the vector spin lies along the velocity.

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}, \quad z = \frac{z_1 + z_2}{2} \tag{6.1}$$

The Schrödinger equation of the center of mass is definite, using the (6.1) coordinates :

$$-i\hbar \frac{\partial \phi}{\partial t} = \frac{1}{2M} \Delta \phi \quad (M = 2m) \tag{6.2}$$

But such a procedure cannot be extended to a pair of Dirac particles, because there is no quantum (and even no classical) relativistic theory of systems of particles. So, de Broglie suggested a formal way easier to generalize. He associated to the particles two different waves ψ and φ, **without making distinction between their coordinates**. So, we have the following equations with the same coordinates x_k:

$$-i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m} \Delta \psi; \quad -i\hbar \frac{\partial \varphi}{\partial t} = \frac{1}{2m} \Delta \varphi \tag{6,3}$$

Now, the **fusion** conditions, expressing **the equality of moment and energy in the case of plane waves**, are :

$$\frac{\partial \psi}{\partial t} \varphi = \psi \frac{\partial \varphi}{\partial t} = \frac{1}{2} \frac{\partial (\psi \varphi)}{\partial t}; \quad \frac{\partial^2 \psi}{\partial x_k^2} \varphi = \frac{\partial \psi}{\partial x_k} \frac{\partial \varphi}{\partial x_k} = \psi \frac{\partial^2 \varphi}{\partial x_k^2} = \frac{1}{4} \frac{\partial^2 (\psi \varphi)}{\partial x_k^2} \tag{6,4}$$

Multiplying the first equation (6,3) by φ and the second by ψ, we find for $\phi = (\varphi \psi)$ the equation (6.2) again. Then, de Broglie **applied the same conditions to all the waves** without restriction to the plane waves, and he applied it to the relativistic case : **it is the « fusion postulate ».**

§2.1) De Broglie's equations of photon.

Consider the Dirac equations of two particles of mass $\frac{\mu_0}{2}$:

$$\begin{aligned} \frac{1}{c} \frac{\partial \psi}{\partial t} &= \alpha_k \frac{\partial \psi}{\partial x_k} + i \frac{\mu_0 c}{2\hbar} \alpha_4 \psi \\ \frac{1}{c} \frac{\partial \varphi}{\partial t} &= \alpha_k \frac{\partial \varphi}{\partial x_k} + i \frac{\mu_0 c}{2\hbar} \alpha_4 \varphi \end{aligned} \tag{6,5}$$

Where $\{\alpha_k, \alpha_4\}$ are the Dirac matrices[3] :

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}; \quad \alpha_k = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \quad (\sigma_k = Pauli\ matrices) \tag{6,6}$$

In analogy with (6,4), de Broglie put the *fusion conditions,* on the Dirac wave-components :

$$\frac{\partial \psi_n}{\partial t} \varphi_m = \psi_n \frac{\partial \varphi_m}{\partial t} = \frac{1}{2} \frac{\partial (\psi_n \varphi_m)}{\partial t}; \quad \frac{\partial \psi_n}{\partial x_k} \varphi_m = \psi_n \frac{\partial \varphi_m}{\partial x_k} = \frac{1}{2} \frac{\partial (\psi_n \varphi_m)}{\partial x_k} \tag{6,7}$$

[3] For he beginning of the theory, we keep the old notations used by de Broglie.

So he found for $\phi = \{\phi_{nm} = \psi_n \varphi_m\}$ a new equation, that **he extended by postulate** to all the ϕ functions even if their form is not $\phi_{nm} = \psi_n \varphi_m$:

$$\boxed{\begin{array}{l}\frac{1}{c}\frac{\partial\phi}{\partial t} = a_k \frac{\partial\phi}{\partial x_k} + i\frac{\mu_0 c}{\hbar} a_4 \phi \\ \frac{1}{c}\frac{\partial\phi}{\partial t} = b_k \frac{\partial\phi}{\partial x_k} + i\frac{\mu_0 c}{\hbar} b_4 \phi\end{array}} \qquad (6,8)$$

The matrices a and b are defined as:

$$\boxed{\begin{array}{l}a_r = \alpha_r \times I, \quad (a_r)_{ik,lm} = (\alpha_r)_{il}\,\delta_{km} \\ b_r = I \times \alpha_r, \quad (b_r)_{ik,lm} = (-1)^{r+1}(\alpha_r)_{km}\,\delta_{il}\end{array}} \; (r = 1,2,3,4) \qquad (6,9)$$

They separately verify the relations of the Dirac matrices ; a and b commute :

$$a_r a_s + a_s a_r = 2\delta_{rs}; \quad b_r b_s + b_s b_r = 2\delta_{rs}; \quad a_r b_s - b_s a_r = 0 \qquad (6,10)$$

owing to which it is easy to prove that the components of ϕ obey the Klein-Gordon equation.

The equations (6,8) with the definitions (6,9) are the *de Broglie photon equations* and we shall see that they include the Maxwell equations.

§2.2) Previously, we must examine some other representations of the photon equations :

"quasi - Maxwellian" form.

First of all, it must be noted that there are too much equations in (6,8) : 32 equations for only 16 components of the wave ϕ. There is a problem of compatibility. To solve the problem, de Broglie added and substracted the two systems in (6,8) :

$$(A) \quad \frac{1}{c}\frac{\partial\phi}{\partial t} = \frac{a_k + b_k}{2}\frac{\partial\phi}{\partial x_k} + i\frac{\mu_0 c}{\hbar}\frac{a_4 + b_4}{2}; \quad (B) \quad 0 = \frac{a_k - b_k}{2}\frac{\partial\phi}{\partial x_k} + i\frac{\mu_0 c}{\hbar}\frac{a_4 - b_4}{2}\phi \qquad (6,11)$$

Further, it will be shown that (6,8) contains exactly the Maxwell equations (up to mass terms), but (6,11) is already an outline of these equations, because this system is divided into a group (A) of "evolution equations" that looks as the Maxwell equations in $\partial\mathbf{E}/\partial t$ and $\partial\mathbf{H}/\partial t$, and a group ($B$) of "condition equations", of the same kind as $div\mathbf{E} = 0$ and $div\mathbf{H} = 0$. In the first paper of 1934 (*Broglie 4*), de Broglie gave only the group (A), but it is easy to prove, in analogy with the Maxwell equations, that:

1) Owing to (6,10), (B) is a consequence of (A).
2) Actually, (B) is only satisfied by the solutions of (A) whose Fourier expansion does not contain a zero frequency. But the zero frequencies are automatically absent from the solutions of (A) if $\mu_0 \neq 0$.
3) Therefore, *iff* $\mu_0 \neq 0$, the condition (B) is a consequence of the evolution equations (A).

- Canonical form.

The equations (6,8) can be transformed in another way :

$$(C)\quad \frac{1}{c}\frac{a_4+b_4}{2}\frac{\partial\phi}{\partial t}=\frac{b_4a_k+a_4b_k}{2}\frac{\partial\phi}{\partial x_k}+i\frac{\mu_0 c}{\hbar}a_4b_4\phi$$

$$(D)\quad \frac{1}{c}\frac{a_4-b_4}{2}\frac{\partial\phi}{\partial t}=\frac{b_4a_k-a_4b_k}{2}\frac{\partial\phi}{\partial x_k} \qquad (6,12)$$

This new system is at the basis of the lagrangian derivation of the theory and of its tensorial form and it was used by de Broglie to *quantise* the photon field and to describe the photon-electron interaction (*Broglie 8*). Just as in (6,11), (*D*) is a consequence of (*C*) *iff* $\mu_0 \neq 0$, which is proved by applying to (*C*) the operator : $\frac{1}{c}\frac{a_4-b_4}{2}\frac{\partial}{\partial t}$ taking into account (6,10). It is noteworthy that the strongest arguments in favor of a massive-photon are not the answers to particular experimental objections but the arguments imposed by the fusion theory, which are linked to the very structure of the theory.

§2. 3) Introduction of a square-matrix wave function.

Now, let us go back to the initial system (6,8), but in terms of relativistic coordinates $x_k=(x,y,z), x_4=ict$ with γ matrices ($\mu,\nu=1,2,3,4$) :

$$\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\delta_{\mu\nu};\quad \mu,\nu=1,2,3,4;\quad \gamma_k=i\alpha_4\alpha_k;\quad \gamma_4=\alpha_4;\quad \gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4 \qquad (6,13)$$

Multiplying (6,8) by $i\alpha_4$, we find owing (6,9) a **new system**, which is not written in terms of a 16 lines column wave function ϕ but **in terms of a 4 × 4 square-matrix** wave function ψ .

$$\partial_\mu\gamma_\mu\Psi-\frac{\mu_0 c}{\hbar}\Psi=0$$

$$\partial_\mu\Psi\tilde{\gamma}_\mu-\frac{\mu_0 c}{\hbar}\Psi=0 \qquad \left(\mu,\nu=1,2,3,4;\quad \tilde{\gamma}_\mu=\gamma_\mu\ transp.\right) \qquad (6,14)$$

The transposed matrices $\tilde{\gamma}$ are easily eliminated because, if two sets of Dirac matrices γ_μ and $\tilde{\gamma}_\mu$, verify the relations (6,13), **there are two and only two non singular matrices Λ and Γ, such that**:

$$\tilde{\gamma}_\mu=\Lambda\gamma_\mu\Lambda^{-1};\ \tilde{\gamma}_\mu=-\Gamma\gamma_\mu\Gamma^{-1};\ \Lambda=\Gamma\gamma_5;\ \mu=1,2,3,4 \qquad (6,15)$$

γ_5 is given in (6,13), and (6,15) is true for $\tilde{\gamma}_\mu$ transposed from γ_μ ; a solution is :

$$\Gamma=-i\gamma_2\gamma_4;\quad \Lambda=\Gamma\gamma_5=-i\gamma_3\gamma_1 \qquad (6,16)$$

The Λ case in (6,15) was given by (*Pauli 1*), and the Γ case was given by de Broglie to eliminate $\tilde{\gamma}_\mu$ in (6,16). Indeed, introducing Γ into (6,14), we find the **system given by Tonnelat, de Broglie, and Pétiau** *(Tonnelat 2 , Broglie 8):*

$$\partial_\mu\gamma_\mu(\psi\Gamma)-\frac{\mu_0 c}{\hbar}(\psi\Gamma)=0$$

$$\partial_\mu(\psi\Gamma)\gamma_\mu+\frac{\mu_0 c}{\hbar}(\psi\Gamma)=0 \qquad (6,17)$$

The equations obtained by substituting Λ to Γ were given recently (*Lochak 3 & following*) **:**

$$\partial_\mu \gamma_\mu (\psi\Lambda) - \frac{\mu_0 c}{\hbar}(\psi\Lambda) = 0$$
$$\partial_\mu (\psi\Lambda)\gamma_\mu - \frac{\mu_0 c}{\hbar}(\psi\Lambda) = 0 \qquad (6,18)$$

The apparently small formal difference (a minus sign) between the two systems (6,17) and (6,18) entails a great physical difference because the solutions of (6,17) and (6,18), exchange between themselves by a mutiplication by γ_5 : **they are dual in space-time and we shall prove that it signifies the exchange between electric and magnetic charges.**

So, the substitution of Λ to Γ in the representation by square matrices of the initial de Broglie's equations (6,5) gives **2 kinds of photons :**

§3) Electric and magnetic photons.

§3.1) The electromagnetic formulae of the photon equations.

The fundamental electromagnetic formulae were given by de Broglie in his first papers, starting from (6,8) (*Broglie 4,5*,8). For the sake of simplicity we start from (6,17) and (6,18) applying a procedure suggested in : (*M.A. Tonnelat*) and then used in : (*Broglie 4*).

Let us expand a 4×4 matrix Θ on the Clifford algebra in $\mathbb{R}^{+---}$:

$$\Theta = I\varphi_0 + \gamma_\mu \varphi_\mu + \gamma_{[\mu\nu]}\varphi_{[\mu\nu]} + \gamma_\mu \gamma_5 \varphi_{\mu 5} + \gamma_5 \varphi_5 \qquad (6,19)$$

φ_0 is a scalar, φ_μ a polar vector, $\varphi_{[\mu\nu]}$ an antisymmetric tensor of rank two, $\varphi_{\mu 5}$ an axial vector (the dual of an antisymmetric tensor of rank three) and φ_5 a pseudo-scalar (the dual of an antisymmetric tensor of rank four). These expressions correspond in $\mathbb{R}^3$ to : a scalar I_1 ; the Lorentz potentials **A,** V (linked to the electric charges) ; the electromagnetic fields : **H, E** ; the pseudo potentials **G**, W (linked to magnetic charges) ; a pseudo-scalar I_2 [4] :

$$\mathbf{H} = Kk_0\left(\varphi_{[23]},\varphi_{[31]},\varphi_{[12]}\right); \mathbf{E} = Kk_0\left(i\varphi_{[14]},i\varphi_{[24]},i\varphi_{[34]}\right)$$
$$\mathbf{A} = K\left(\varphi_1,\varphi_2,\varphi_3\right); iV = K\varphi_4$$
$$-i\mathbf{G} = K\left(\varphi_{[15]},\varphi_{[25]},\varphi_{[35]}\right); W = K\varphi_{[45]} \qquad (6,20)$$
$$I_1 = K\varphi_0; iI_2 = K\varphi_5 \quad \left(k_0 = \frac{\mu_0 c}{\hbar}; K = \frac{\hbar}{2\sqrt{\mu_0}}\right)$$

Now, if we develop (6,17) and (6,18) owing to (6,19) and (6,20), we find two sets of equations.

§3.2) The equations of the "electric photon" (Γ matrix).

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[4] With respect to our preceding publications, **B** is changed in **G** to avoid a confusion with the induction. So that, the Lorentz **polar quadripotential** linked to the electric Einstein photon remains (V, **A**) while the **pseudo-quadripotential** linked to the **magnetic photon** is (W, **G**).

The expansion of the matrix wave-function $\Psi = \psi\,\Gamma$ according to (6,19) splits the equation (6,17) into two systems (*Broglie 8, 9*), that we call now the « **electric photon** » for reasons given below:

$$\text{(M)}\qquad \begin{pmatrix} -\dfrac{1}{c}\dfrac{\partial \mathbf{H}}{\partial t} = rot\mathbf{E}; \quad \dfrac{1}{c}\dfrac{\partial \mathbf{E}}{\partial t} = rot\mathbf{H} + k_0^2\mathbf{A} \\ div\mathbf{H} = 0; \quad div\mathbf{E} = -k_0^2 V \\ \mathbf{H} = rot\mathbf{A}; \quad \mathbf{E} = -\mathbf{grad}V - \dfrac{1}{c}\dfrac{\partial \mathbf{A}}{\partial t}; \quad \dfrac{1}{c}\dfrac{\partial V}{\partial t} + div\mathbf{A} = 0 \end{pmatrix} \qquad (6,21)$$

$$\text{(NM)}\qquad \begin{pmatrix} -\dfrac{1}{c}\dfrac{\partial I_2}{\partial t} = k_0 W; \quad \mathbf{grad}I_2 = k_0\mathbf{G}; \quad \dfrac{1}{c}\dfrac{\partial W}{\partial t} + div\mathbf{G} = k_0 I_2 \\ rot\mathbf{G} = 0; \quad \mathbf{grad}W + \dfrac{1}{c}\dfrac{\partial \mathbf{G}}{\partial t} = 0; \quad \left\{\left(k_0 I_1 = 0;\ k_0 \neq 0\right) \Rightarrow I_1 = 0\right\} \end{pmatrix} \qquad (6,22)$$

Actually, de Broglie fixed his attention essentially on the first system of equations (6,21), that he denoted : (M) ("Maxwellian"), for evident reasons, and he considered it as the **equations of the photon (M : spin 1). It was the great victory of his theory : the deduction of Maxwell's equations from Dirac's equation**.

Curiousely, de Broglie was rather puzzled by the second system **spin 0**, that he named negatively : **NM** ("Non maxwellian"), without giving any clear interpretation. He thought at first of a meson, but then abandoned the idea. Here, we shall adopt the following very simple interpretation :

It is quite natural to find two systems of equations because the fundamental equations (6,8) are not the equations of a particle of spin 1, but of a particle of **maximum spin 1** : a combination of two particles of spin ½, as de Broglie underlined it. For this reason, just as for a diatomic molecule, we find two states described by two systems of equations : an **orthostate** of spin 1=1/2+1/2 (parallel spins) and a **parastate** of spin 0 =1/2-1/2 (opposite spins) : we shall adopt this interpretation.

Both states have equal rights with respect to the symmetry laws, because both are linked by a symmetry of form and both have a physical sense, despite that one of them (the orthostate : spin 1) is related to a far more celebrated case : the Maxwell equations, while the other (the para state : spin 0) is related to the « smaller » Aharonov-Bohm effect, as it will be shown later.

Thus we have two photons, more precisely two spin states : 1 and 0, of a photon described by the systems (6,21) and (6,22). And it is not a « general » photon, but only an **electric photon, because we shall find another one : a magetic photon.** By the moment, we have just the electric photon with two photon states : a spin 1 state : (M), " Maxwellian" and a spin 0 state : (NM), "Non-Maxwellian".

a) The (M) equations are Maxwell's equations with two differences :

1) The first difference is the presence of the *mass terms,* which introduces a link between fields and potentials, the latter becoming physical quantities and they lose their gauge invariance.

2) The second difference is the automatic definitions of the fields, through the Lorentz potentials with the Lorentz gauge condition :

$$\mathbf{H} = rot\mathbf{A}; \quad \mathbf{E} = -\mathbf{grad}V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}; \quad \frac{1}{c}\frac{\partial V}{\partial t} + div\mathbf{A} = 0 \qquad (6,23)$$

These relations are not arbitrary added to the field-equations, as they were in the classical theory : **they appear automatically and they are themselves field-equations**, as a consequence of the massive photon. Of course, they were already present in a hidden form in (6,8), (6,11), (6,12), (6,17).

As a consequence of (6,21), the fields and potentials do not obey the d'Alembert wave equation but the Klein-Gordon equation:

$$\square F + k_0^2 F = 0; \quad \left(F = \mathbf{E}, \mathbf{H}, \mathbf{A}, V, \mathbf{G}, W, I_1, I_2\right) \qquad (6,24)$$

The electrostatic solution is not the Coulomb potential $\frac{1}{r}$ but the Yukawa potential $V = \frac{e^{-r/k_0}}{r}$ which remains a long range potential because of the smallness of the Compton wave number $k_0 = \frac{\mu_0 c}{\hbar}$.

b) The (NM) equations were previousely considered by de Broglie (as it was said before) as describing an independant spin 0 **meson** with a far greater μ_0 mass than the mass of the photon. Which is astonishing as far as the equations (M) and (NM) came from the decomposition of the same system of equations, so that both rest masses are obliged to be equal ![5] Of course, we shall abandon this idea, which was actually later forgoten by de Broglie himself. Our interpretation will be based, on the contrary, on the link between the two systems (M) and (NM), admitted as a fact.

The system (NM) decribes a chiral particle because : I_1 is a true invariant, but $I_1 = 0$; actually, the particle is definite by the the second invariant I_2 which is a pseudo – invariant, dual of an antisymmetric tensor in $\mathbb{R}^{+---}$ (with $I_2 \neq 0$), and by the pseudo – quadrivector $\left(\mathbf{G}, W\right)$ in $\mathbb{R}^{+---}$.

It must be noted that de Broglie remarked *(Broglie 9)* that the situation could be interpreted in another way, defining a second electromagnetic field (he said : an "anti-field") which *equals zero* in virtue of (6,22) :

$$\mathbf{H}' = \frac{1}{c}\frac{\partial \mathbf{G}}{\partial t} + \mathbf{grad}W; \quad \mathbf{E}' = rot\mathbf{G} \qquad (6,25)$$

We shall follow the second interpretation, on the basis of a symmetry between electricity and magnetism developed in our papers concerning the photon (*Lochak 20*) and the magnetic monopole (*Lochak 5, Lochak 10*)[6]. We consider the systems (6,21) - (6,22) as simply describing, with « equal rights » ; the orthohydrogene state (spin 1) and the parastate (spin 0) of an **electric photon**, for the following reasons :

1) In the system (M) we have an electromagnetic field : $\left(\mathbf{E}, \mathbf{H}\right)$ and a *polar* 4-potential : $\left(V, \mathbf{A}\right)$ related to $\left(\mathbf{E}, \mathbf{H}\right)$ by the Lorentz formulae (6,23). These fields and potentials enter in the dynamics of an *electric* charge. Because $k_0 \neq 0$, we have in general : $div\mathbf{E} \neq 0$; so that, the electric field $\mathbf{E}$ is not transversal, contrary to the magnetic field $\mathbf{H}$: $\mathbf{E}$ has a small longitudinal component, of the order of k_0.

2) In the (NM) equations, we have a *pseudo* invariant I_2 and an *axial* 4-potential $\left(\mathbf{G}, W\right)$, to which may be added the invariant I_1 and the "anti-field" $\{\mathbf{E}', \mathbf{H}'\}$, defined in (6,25), and which will be related to

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[5] This strange idea was probably inspired by the just happening discovery of the Yukawa meson.

[6] The de Broglie definition (6,25) of $\mathbf{H}'$ and $\mathbf{E}'$ in terms of a *pseudo* quadripotential $\left(\mathbf{G}, W\right)$, was later rediscovered by (*Cabibbo & Ferrari*).

magnetism. But here : $I_1 = \mathbf{E}' = \mathbf{H}' = 0$, which confirms the electric character of the (NM) photon, by the annihilation of magnetic quantities.

The difference between the de Broglie interpretation and mine is that now (NM) is no more separated from the spin 1 (state M) : it is the spin 0 state of the same photon. The electric photon is the whole system (6,21) - (6,22) with two values of spin.

§3.3) The equations of the magnetic photon $(\Lambda$ matrix$)$.

This second photon is given by (6,18) with $\Lambda = \Gamma\gamma_5$ intead of Γ in (6,17)**.** The primed new field-components are the *dual* of the preceding ones, which means that **the matrix γ_5 exchanges electricity and magnetism** *(Lochak 5, 10)* :

$$\text{(M)}\qquad \begin{pmatrix} -\frac{1}{c}\frac{\partial \mathbf{H}'}{\partial t} = rot\mathbf{E}' + k_0^2\mathbf{G}'; \quad \frac{1}{c}\frac{\partial \mathbf{E}'}{\partial t} = rot\mathbf{H}' \\ div\mathbf{H}' = k_0^2 W'; \quad div\mathbf{E}' = 0 \\ \mathbf{H}' = \mathbf{grad}W' + \frac{1}{c}\frac{\partial \mathbf{G}'}{\partial t}; \quad \mathbf{E}' = rot\mathbf{G}'; \quad \frac{1}{c}\frac{\partial W'}{\partial t} + div\mathbf{G}' = 0 \end{pmatrix} \qquad (6,26)$$

$$\text{(NM)}\qquad \begin{pmatrix} -\frac{1}{c}\frac{\partial I_1}{\partial t} = k_0 V'; \quad \mathbf{grad}I_1 = k_0\mathbf{A}'; \quad \frac{1}{c}\frac{\partial V'}{\partial t} + div\mathbf{A}' = k_0 I_1 \\ rot\mathbf{A}' = 0; \quad \mathbf{grad}V' + \frac{1}{c}\frac{\partial \mathbf{A}'}{\partial t} = 0; \quad \left\{\left(k_0 I_2 = 0; \quad k_0 \neq 0\right) \Rightarrow I_2 = 0\right\} \end{pmatrix} \qquad (6,27)$$

The new photon is associated, as the former, with a couple of fields. But the situation is inverted:

1) The "anti-field" $(\mathbf{E}',\mathbf{H}')$ and the *axial* 4-potential $(W',\mathbf{G}')$ satisfy the Maxwell-type (M) system (6,26). The definition (6,25) of the "anti-fields" now appears in (6,26) automatically (and not by an *a priori* définition), as one of the field-equations. Now $(\mathbf{E}',\mathbf{H}')$ are not equal to zero. The fields $(\mathbf{E}',\mathbf{H}')$ are exactly those that enter in the dynamics of a magnetic charge: a monopole (*Lochak 5, 10 and here : Chapters 2,3)*.

Besides, "symmetrically" to the electric case, we have now $div\mathbf{H}' \neq 0$, so that, in a plane wave, the *magnetic* field $\mathbf{H}'$ (instead of the electric one $\mathbf{E}'$) has now a small longitudinal component, of the order of k_0, while $\mathbf{E}'$ is transversal. We have a **magnetic photon**.

2) Now, the *polar* potentials $(V',\mathbf{A}')$ dual from $(W',\mathbf{G}')$ appear in the (NM) system, i.e. in the *spin 0 state*. The invariant I'_1 and the pseudo- invariant I'_2 invert their roles : we have now $I'_1 \neq 0$ and $I'_2 = 0$. The electromagnetic field : $(\mathbf{E}',\mathbf{H}')$ defined by the Lorentz formulae (6,23) gives now : $(\mathbf{E}' \neq 0, \mathbf{H}' \neq 0)$ in the maxwellian formulae (M) and $(\mathbf{E}' = \mathbf{H}' = 0)$ in the non maxwellian formulae (NM), conversely to what we had in the electric case.

It is a remarkable fact, that **de Broglie's *fusion* of two Dirac equations do not only gives the classical Maxwell equations as it was proved by de Broglie, but now we prove that it also defines *two classes* of photons, corresponding respectively to electric or magnetic charges. The algebraic symmetry excludes any other possibility.**

The symmetry between the two electromagnetic fields is all the more interesting that such a symmetry already appears in the Dirac equation itself, in the form of *two minimal interactions* corresponding to electric and magnetic charge, associated the two kinds of fields (*Lochak Ch. 2 & 1, ... 10*). Symmetries of Dirac's and de Broglie's equations are thus mutually reinforced. Now we must answer other questions.

-We have 2 kinds of photons : the electric and the magnetic photon.

But is their physical difference given by the difference between the two couples of equations : (2,10) - (2,11) and (6,17) - (6,18) or : (Dirac gauge and equation) and (chiral gauge and equation) ? Yes, because it is the difference between the motion of an electron or a monopole in an electrodynamic field. For instance in a linear electric field, the electron is linearly accelerated, while the monopole rotates around the field. And conversely in a linear magnetic field.

-Actually there are not only two but 4 kinds of photons because they can have a spin 1 or a spin 0.

The preceding answer is only related to the spin 1. We must now answer a new question : are the spin 0 photons already known ? The answer is yes and there is a wellknown example :

§4) The Aharonov-Bohm effect.

Consider the equations of (NM) potentials **:** (6,22) and (6,27) :

1) Spin 0 magnetic photon : $-\frac{1}{c}\frac{\partial I_2}{\partial t} = k_0 W; \quad \mathbf{grad}I_2 = k_0\mathbf{G}; \quad \frac{1}{c}\frac{\partial W}{\partial t} + div\mathbf{G} = k_0 I_2$

2) Spin 0 electric photon : $-\frac{1}{c}\frac{\partial I_1}{\partial t} = k_0 V'; \quad \mathbf{grad}I_1 = k_0\mathbf{A}'; \quad \frac{1}{c}\frac{\partial V'}{\partial t} + div\mathbf{A}' = k_0 I_1$

We must remember that the spin 1 electric photon is associated with a magnetic spin 0 photon by the pseudo – invariant I_2, while the spin 1 magnetic photon is associated with an electric spin 0 photon by the true invariant I_1. The preceding relations immediately imply that the spin 0 potentials are the **gradients of relativistic invariants**, which verifiy the Klein-Gordon equation :

$$\partial_\mu I_1 = k_0 \mathbf{A}_\mu; \quad \Box I_1 + k_0^2 I_1 = 0; \qquad \partial_\mu I_2 = k_0 G_\mu; \quad \Box I_2 + k_0^2 I_2 = 0 \qquad (6,28)$$

We know that in virtue of (6,22) and (6,27), or (6,28) the corresponding electromagnetic fields equal zero. The question is : **how the spin 0 photon can be detected** ? More precisely : since these fieldless potentials are unable to generate a force, what does remain which could be observed ? Of course : the **phase,** first characteristic of a wave. The Aharonov-Bohm effect was imagined at first by David Bohm[7] to answer the question, and to prove that contrary to a common idea, the electromagnetic potentials are not only mathematical intermediates (even if they can play this role) : they are observable physical quantities.

§4.1) The effect.

The idea suggested by Bohm (see : *Aharonov-Bohm, Tonomura, Olariu-Popescu, Lochak 3*) was to modify electron interferences by a fieldless magnetic potential created by a magnetic string or by a thin solenoid orthogonal to the plan of interfering electron trajectories, as it is shown on the Fig. 1. The Young slits are obtained owing to a Fresnel - Möllenstedt biprism.

[7] I know that because I was acquainted with Bohm who lived in Paris in that time.

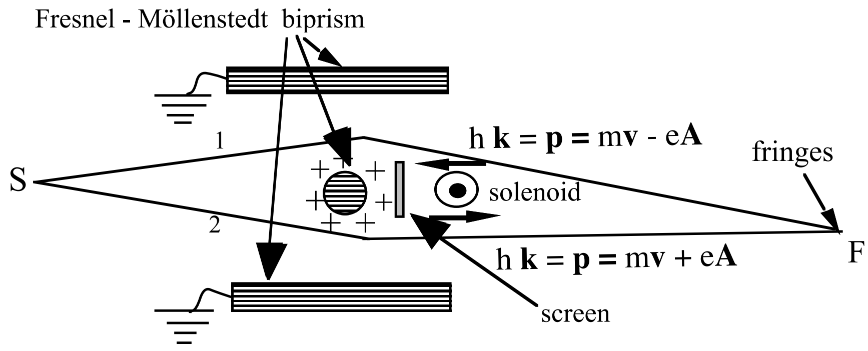

Fig. 1

Aharonov-Bohm experiment

The solenoid must be in principle infinitely long, so that the magnetic field emanating from the extremities cannot perturb the experiment : it is assumed in the calculations, but actually a few millimeters are sufficient because the transverse dimensions of the device are of the order of microns. This arrangement of the solenoid has led to the idea that the magnetic flux through the trajectories quadrilateral plays an essential role. Many disagree with that idea (see : *Lochak 3*).

The problem of eliminating this hypothesis was elegantly solved by Tonomura (see : *Tonomura*) by substituting the rectilinear string by a microscopic torus ($10\mu m$) : one of the electron beams passes through the torus and the other outside, the magnetic lines being trapped in the torus.

Let us give an intuitive interpretation of the experiment. The principle is that the wave vector of an electron in a magnetic potential is given by the de Broglie wave (*Broglie,5*) which is a direct consequence of the identification of the principles of Fermat and of least action ($\mathbf{p}$ is the Lagrange momentum) :

$$\frac{h}{\lambda}\mathbf{n} = h\mathbf{k} = \mathbf{p} = m\mathbf{v} + e\mathbf{A} \tag{6,29}$$

It is obvious on the preceeding formula, that interference and diffraction phenomena are influenced by the presence of a magnetic potential independently of the presence of the field because the interferences depend only on the phase. It is well known in optics : an interference figure is shifted in a Michelson interferometer by introducing a plate of glass in one of the virtual beams, which causes a phase shift and thus a change of the optical path without any additive force.

These phenomena are manifestly **gauge dependent** : if we add something to $\mathbf{A}$, would it be a gradient or not, in the de Broglie wave λ (6,29), the last is modified. This is evident even on the classical de Broglie formula : $\lambda = \frac{h}{mv}$ when $\mathbf{A} = 0$, which is gauge dependent too, a fact often emphasized by de Broglie himself who said : « **If gauge invariance would be general in quantum mechanics, the electron interferences could not exist** ».

In the case of Aharonov-Bohm experiment there is an additive phase on both interfering waves in opposite directions, which doubles the shift of the interference fringes. Let us recall a proof of the effect, independent from the fact that a potential generates forces or not (*Lochak 3*).

§4.2) The magnetic potential of an infinetely thin and infinitely long solenoid.

We consider the case corresponding to the realized Aharonov-Bohm experiment : electrons diffracted on Young slits and falling on a magnetic solenoid orthogonal to the plane electron trajectories, according to the Fig. 1 and, further, to the schematic Fig. 2, the solenoid is along Oz. To simplify the calculations, we shall neglect the photon mass, only important in the symmetry laws which are taken into account in all the formulae ; so that, to omit the photon mass only means to omit negligible corrections.

The **electric charge** of the diffracted electrons implies that they « see » the electromagnetism through the Lorentz potentials $(V, \mathbf{A})$ and thus through the equations (M) : (6,21). These equations derive from the pseudo - invariant I_2. Now, there is an obvious invariant in the Aharonov-Bohm effect : the rotation angle $\varphi = Arctg\, y/x$ around the axis Oz. So we shall write :

$$I_2 = \varepsilon k_0 Arctg(y/x) \qquad (6,30)$$

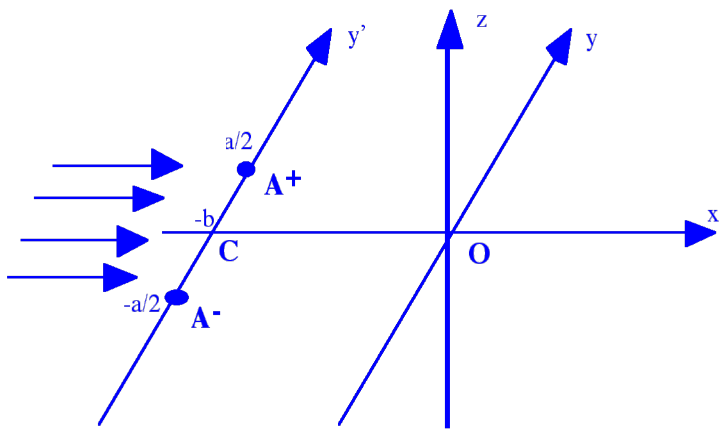

Fig. 2
Aharonov-Bohm scheme

k_0 is the quantum wave number of the photon and ε a convenient dimensional constant, the value of which is not important for our calculation. Neverthless, something seems wrong here, because (y/x) is P-invariant so that, with the definition (6,30), I_2 seems to be a P-invariant too and not a pseudo - invariant, as it needs in (6,22).

But it is not so because (y/x) is P-invariant only in the space $\mathbb{R}^2$: (x,y), not in the space $\mathbb{R}^3$ (x,y,z). In our case, the inversion is the P – transformation : $(x,y,z) \to (-x,-y,-z)$, which implies the inversion of Oz and thus of the angle φ. So that (y/x) is really a pseudo – invariant in $\mathbb{R}^3$.

Thus we have, in virtue of (6,22):

$$\mathbf{grad} I_2 = k_0 \mathbf{G} \qquad (6,31)$$

$$\mathbf{G}_x = -\varepsilon \frac{y}{x^2 + y^2} \,; \quad \mathbf{G}_y = \varepsilon \frac{x}{x^2 + y^2} \,; \quad \mathbf{G}_z = 0 \qquad (6,32)$$

§4.3) The theory of the effect

The commonly admitted theories are useless complicated (*Olariu S., Iovitsu Popescu*). For the physical bases of the effect, the best is to start from the brillant book : (*Tonomura*). To find the formula of fringes it is sufficient to take the geometrical optics approximation with the phase $\varphi = S/\hbar$ of de Broglie's wave and the principal Hamilton function S obeying the Hamilton-Jacobi equation with the potential (6,32) :

$$2m\frac{\partial S}{\partial t} = \left(\frac{\partial S}{\partial x} + \varepsilon\frac{y}{x^2+y^2}\right)^2 + \left(\frac{\partial S}{\partial y} - \varepsilon\frac{x}{x^2+y^2}\right)^2 \qquad (6,33)$$

The electronic wave propagates from $x = -\infty$ to $x = +\infty$ and the Young slits A^+ and A^- (Fig. 2) are on a parallel to Oy, at a distance $\pm\frac{a}{2}$ from the point C located at $x = -b$.

The pseudo-potential $\mathbf{G}$ appearing in (6,30) and (6,31) is the gradient of I_2, so that $\mathbf{G}$ and I_2 satisfy up to μ_0 the equations (NM), (6,27). They are independant of t because $W = 0$.

The equation (6,33) is immediately integrated, defining the phase :

$$\Sigma = S - \varepsilon\, Arctg\, y/x \qquad (6,34)$$

Which gives :

$$2m\frac{\partial \Sigma}{\partial t} = \left(\frac{\partial \Sigma}{\partial x}\right)^2 + \left(\frac{\partial \Sigma}{\partial y}\right)^2 \qquad (6,35)$$

Chosing a complete integral of (6,35) and thus of (6,32), owing to (6,33), we have :

$$\Sigma = Et - \sqrt{2mE}\left(x\cos\theta_o + y\sin\theta_o\right) \qquad (6,36)$$

$$S = Et - \sqrt{2mE}\left(x\cos\theta_o + y\sin\theta_o\right) + \varepsilon\, Arctg\frac{y}{x} \qquad (6,37)$$

Or, in polar coordinates $x = r\cos\theta$, $y = r\sin\theta$:

$$S = Et - \sqrt{2mE}\, r\cos(\theta - \theta_o) + \varepsilon\,\theta \qquad (6,38)$$

The Jacobi theorem gives the trajectories (**the wave rays**) :

$$\frac{\partial S}{\partial \theta_o} = \sqrt{2mE}\left(x\sin\theta_o - y\cos\theta_o\right) = \mu \; ; \quad \frac{\partial S}{\partial E} = t - \sqrt{\frac{m}{2E}}\left(x\cos\theta_o + y\sin\theta_o\right) = t_o \qquad (6,39)$$

Finally, with[8] $E = \frac{1}{2}m\mathrm{v}^2$ we have the motion :

$$x\cos\theta_o + y\sin\theta_o = \mathrm{v}\left(t - t_o\right) \qquad (6,40)$$

We see that the *rays* (electron trajectories), defined in (6,39) are **orthogonal to the moving planes but they are not orthogonal to the equal phase surfaces** (6,37) – (6,38) except far from the magnetic string $(x \to \infty)$, when the potential term of the order of ε becomes negligible.

[8] We are obviously far from relativity.

Therefore, despite the presence of a potential, the electronic trajectories remain rectilinear and are not deviated, because the magnetic field equals zero in virtue of (6,22). **The velocity** $\mathrm{v} = Const$ **remains the one of the incident electrons because of the conservation of energy.**

But the diffraction of waves through the slits A^{+} and A^{-} creates, for the electron trajectories, an interval of possible angles θ_o, equal the angles of the interference fringes, modified by the magnetic potential :

There is no deviation of the electrons, only a deviation of the angles of phase synchronization between the waves issued from A^{+} and A^{-}. This is the Aharonov-Bohm effect, which is in accordance with the definition of the spin 0 photon (6,22).

It would be useless to reproduce the end of the theory of Aharonov-Bohm effect (*see for instance Lochak 3*). Let us only recall the total phase-shift :

$$\Delta\varphi = \frac{\Delta S}{h} = \frac{a\theta_o}{\lambda} + \frac{2\varepsilon\xi}{h} \qquad (6,41)$$

The first term gives the standard Young fringes (the notations are those of Fig. 2), **the second term is the Aharonov-Bohm effect** : $\xi = Arctg\dfrac{a}{2b}$ equal to half the angle under which the Young slits are seen from the solenoid, which entails a dependence of the effect on the position of the string : one can assert that **the effect decreases when the distance b increases.**

We see that the theory of the Aharonov-Bohm effect is a simple consequence of the definition of the invariant in the system (6,27), as the invariant rotation angle around the axis of he solenoid.

§5) Conclusions on the theory of light.

We suggest a new theory of light based on 4 photons.

1) At first the Einstein photon known in optics from 1905, and later identified by de Broglie (1922) as a **vectorial spin 1** particle, which we call here the **electric photon,** because it interacts with the electric charges (principally with electrons)
2) **A pseudovectorial spin 1 magnetic photon**, analogous to the electric Einstein photon : it appears in the theory of **leptonic magnetic monopole**s (*see : Ch.2,3 and Lochak*). The magnetic photon plays in the physics of monopoles a role exactly similar to the role played by the electric photon in the theory of electrons.
3) **Two spin 0** photons (one **electric** and the other **magnetic**), related to 2 classes of respectively electric and magnetic fieldless phenomena ; an example is the Aharonov-Bohm effect.
4) It must be added that **in the 4 photons theory of light there are two Maxwell displacements** : an electric displacement and a magnetic displacement. Let us recall what is the Maxwell displacement[9] : at the beginning, he tried to unify the electromagnetism on the basis several fundamental laws : the laws of Coulomb : $\nabla.\mathbf{E} = 4\pi\rho$, Ampere : $\nabla\times\mathbf{H} = \dfrac{4\pi}{c}\mathbf{J}$ and Faraday : $\nabla\times\mathbf{E} + \dfrac{1}{c}\dfrac{\partial\mathbf{H}}{\partial t} = 0$. But he found between them an incoherence because the third law depends of time and the other ones not. The fact was well known and was objected to Faraday ; but the critics of Maxwell went contrary to the unanimity of physicists : he considered the law of

[9] See the excellent Chapter 6 of (*Jackson*). Our formalism is different from Jackson's because here we are in the domain of quantum laws which are written in the vacuum.

Faraday as the right one and he decided to introduce a time dependence in the other two laws. He replaced the Coulomb law by a continuity law : $\nabla . \mathbf{J} + \frac{\partial \rho}{\partial \mathbf{t}} = 0$, owing which the Ampere law became : $\nabla \times \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{\mathbf{4}\pi}{c}\mathbf{J}$. So, he found the celebrated Maxwell equations in which appeared wave like solutions from which Maxwell found the **electromagnetic theory of light** and which later gave rise to the **radio-waves**.

If we compare the (M) equations (6.21) of the electric photon with our (M) equations (6.26) of the magnetic photon, the analogy is evident : the terms of a Maxwell displacement are present in the magnetic photon and it may be supposed that they are able to analogous physical consequences involving magnetic monopoles instead of electrons : the Aharonov-Bohm effect is a first example.

Now there is another fact which is true for 70 years, without being pointed out until now, as far as I know. It is the fact that **the de Broglie theory, based on the principle of fusion,** implies automatically the **displacement** previousely introduced by Maxwell through an external argument**.** The fact is hidden because the Maxwell equations make now a unit often abridged in different algebraic forms, while the displacements are more or less forgotten or rejected in the subtelties of history of Science. Despite that the postulate of fusion has an algebraic character, it has the advantage of the unicity and of being a direct bridge between the problem of electromagnetism and the Dirac equation of the electron, the stronger equation of quantum mechanics.

It must be added that the de Broglie theory of the photon considered as a composite particle, gave rise to an extension to a general theory of spin particles, including the gravitaton : see the second part of the present Chapter. Already in the present first part we have seen several generalisations of de Broglie's theory of light, as the magnetic photon linked to the magnetic monopole, and the Aharonov - Bohm effect, which gives rise to a new domain of electrodynamical phenomena.

The here sugested theory of light is a generalisation of Broglie's theory of light, with electric and magnetic photons. A new hypothesis of the present theory is that the spin 0 is condidered as a state of the photon with the same rights as the spin 1 : there are not only two kinds of spin-1 photons but also of two spin-0 photons. In other words, the photon world is divided in the same two categories as other composite quantum objects. There are **orthophotons of spin 1 and paraphotons of spin 0,** just as there is orthohydrogen and parahydrogen. But concerning the photon, it is a new idea, contrary to the case of orthohydrogen and parahydrogene, known for almost a century. This is why many questions still remain asked :

- What happens with the spin - 0 photons in the thermodynamical equilibrium ?
- We have seen that paraphotons being fieldless, are unable to create a force ; so, are they able to produce something like a photoelectric effect ? It seems that not.
- More generally, are they true quantum wave-particle objects, or « pure - phases », pure potentials without particles ? (I beg the pardon of my old Master Louis de Broglie !)
- There are arguments in favour of some of these hypotheseses. For instance the existence of a magnetic spin 1 photon is confirmed by the experiments on the leptonic monopole (see *Urutskoiev et al.*)
- Until now, the Aharonov-Bohm effect was a remarkable, but isolated orphan effect. Here it is integrated in a general theory. This is well, but a question remains : is this effect exceptional, or is it a sample of a « class » of new phenomena ? The equations define mathematically such a class but it must be experimentally proved that such phenomena really exist as physical effects.

§6) Hamiltonian, lagrangian, current, energy, spin.

§6.1) The lagrangian.

Now, let us go back to the 16 lines column wave function and the canonical form (6,12), keeping only (C), because (D) is deduced from it:

$$\frac{1}{c}\frac{a_4+b_4}{2}\frac{\partial\phi}{\partial t}=\frac{b_4a_k+a_4b_k}{2}\frac{\partial\phi}{\partial x_k}+i\frac{\mu_0 c}{\hbar}a_4b_4\phi \tag{6.42}$$

Note the presence, of $\left(a_4+b_4\right)/2$ in factor of ∂/∂t, which seems unescapable to obtain coherent definitions for tensor densities. The hamiltonian operator is :

$$H=i\hbar\left[\frac{b_4a_k+a_4b_k}{2}\frac{\partial}{\partial x_k}+i\frac{\mu_0 c}{\hbar}a_4b_4\right] \tag{6.43}$$

And the lagrangian density is (with $\phi^+=\phi\left(h.c.\right)$) :

$$L=-i\hbar c\left[\phi^+\left(\frac{1}{c}\frac{a_4+b_4}{2}\frac{\partial\phi}{\partial t}-\frac{b_4a_k+a_4b_k}{2}\frac{\partial\phi}{\partial x_k}-i\frac{\mu_0 c}{\hbar}a_4b_4\phi\right)+c.c.\right] \tag{6.44}$$

§6.2) The current density vector.

The general formula :

$$J_\mu=\frac{i}{\hbar}\left[\frac{\partial L}{\partial\phi_{,\mu}}\phi-\frac{\partial L}{\partial\phi^+_{,\mu}}\phi^+\right] \tag{6.45}$$

gives, with (6.44) :

$$J_k=-c\phi^+\frac{b_4a_k+a_4b_k}{2}\phi;\quad J_4=ic\rho;\quad \rho=\phi^+\frac{a_4+b_4}{2}\phi \tag{6.46}$$

Therefore $\int\rho\, dv$ *is not definite-positive.* But on the other hand, we shall find a *definite-energy :* $\int\rho W\, dv\geq 0$, contrary to what happens in the Dirac electron. This result will be generalized in the general theory of particles with $spin=\frac{n}{2}$.

In terms of electromagnetic quantities, (6.45) is given by the Geheniau formulae with two kinds of terms corresponding to spin 1 and spin 0 in the case of an *electric photon* (*Broglie 9*). Here, until the end of the § 6.3, we give only the translation of the formulae in the electric case (they were not translated until now in the magnetic case) :

$$\begin{aligned}\mathbf{J}&=\frac{i}{\hbar c}\left[\mathbf{A}^*\times\mathbf{H}+\mathbf{H}^*\times\mathbf{A}+V^*\mathbf{E}-\mathbf{E}^*V\right]+\frac{c}{4}\left(I^*_2\,\mathbf{G}+\mathbf{G}^*I_2\right)\\ \rho&=\frac{i}{\hbar c}\left[\left(\mathbf{A}^*.\mathbf{E}\right)-\left(\mathbf{E}^*.\mathbf{A}\right)\right]+\frac{1}{4}\left(I^*_2\,W+W^*I_2\right)\end{aligned} \tag{6.47}$$

For the energy tensor, we have the general formula :

$$T_{\mu\nu} = -\frac{\partial L}{\partial \phi_{,\mu}}\phi_{,\mu} - \frac{\partial L}{\partial \phi^+_{,\mu}}\phi^+_{,\mu} + L\delta_{\mu\nu}, \tag{6.48}$$

with the lagrangian (6,44), which gives:

$$\begin{aligned}
T_{ik} &= -\frac{i\hbar c}{2}\left[\phi^+ \frac{b_4 a_k + a_4 b_k}{2}\frac{\partial \phi}{\partial x_k} + \frac{\partial \phi^+}{\partial x_k}\frac{b_4 a_k + a_4 b_k}{2}\phi\right] \\
T_{i4} &= \frac{\hbar}{2}\left[\phi^+ \frac{b_4 a_k + a_4 b_k}{2}\frac{\partial \phi}{\partial t} + h.c.\right]; \quad T_{4i} = -\frac{\hbar c}{2}\left[\phi^+ \frac{a_4 + b_4}{2}\frac{\partial \phi}{\partial x_i} + h.c.\right] \\
T_{44} &= -w = i\hbar \frac{i\hbar}{2}\phi^+ \frac{a_4 + b_4}{2}\frac{\partial \phi}{\partial t} = -\phi^+ H \phi
\end{aligned} \tag{6.49}$$

In the electromagnetic form, we have :

$$\begin{gathered}
T_{\mu\nu} = \frac{1}{2}\left(F_{\mu\lambda}\frac{\partial \mathbf{A}_\lambda}{\partial x_\nu} - \mathbf{A}_\lambda \frac{\partial F_{\lambda\mu}}{\partial x_\nu}\right) - \frac{i\hbar}{8}\left(I*_2 \frac{\partial \mathbf{G}_\lambda}{\partial x_\nu} + \mathbf{G}*_\mu \frac{\partial I_2}{\partial x_\nu}\right) + \text{c.c.} \\
\text{where} : F_{\mu\lambda} = \frac{\partial \mathbf{A}_\mu}{\partial x_\nu} - \frac{\partial \mathbf{A}_\nu}{\partial x_\mu}
\end{gathered} \tag{6.50}$$

In particular, the energy density ρW takes the form:

$$\begin{aligned}
T_{44} &= \frac{1}{2c}\left[\left(\mathbf{A}*.\frac{\partial \mathbf{E}}{\partial t}\right)\left(\mathbf{E}*.\frac{\partial \mathbf{A}}{\partial t}\right) + \left(\mathbf{A}.\frac{\partial \mathbf{E}*}{\partial t}\right)\left(\mathbf{E}.\frac{\partial \mathbf{A}*}{\partial t}\right)\right] \\
&+ \frac{i\hbar c}{2}\left[\left(I_2 * \frac{\partial W}{\partial t}\right)\left(W*\frac{\partial I_2}{\partial t}\right) - \left(I_2 \frac{\partial W*}{\partial t}\right)\left(W\frac{\partial I_2 *}{\partial t}\right)\right]
\end{aligned} \tag{6.51}$$

The tensor $T_{\mu\nu}$ is often symmetrized, putting : $T_{(\mu\nu)} = \frac{1}{2}\left(T_{\mu\nu} + T_{\nu\mu}\right)$, but there are strong arguments in favor of the nonsymmetric tensor (*Costa de Beauregard 1 and Broglie 9*).

And we can find other tensors, the integrals of which are equal to the integral of the precedings (they differ by a divergence). One of these tensors is[10]:

$$M_{ik} = M_{ki} = \mu_0 c^2 \phi^+ \frac{a_i b_k + a_k b_i}{2}\phi;\ M_{i4} = M_{4i} = -\mu_0 c^2 \phi^+ \frac{a_i + b_i}{2}\phi;\ M_{44} = \mu_0 c^2 \phi^+ \phi;\ \left(i,k = 1,2,3\right) \tag{6.52}$$

This is a tensor of Maxwell type because we find, in electromagnetic terms, for the electric photon:

$$M_{i4} = \left(\mathbf{E}.\mathbf{H}*\right)_i + \left(\mathbf{E}*.\mathbf{H}\right)_i - k_0^2\left(V*\mathbf{A}_i + V\mathbf{A}*_i\right);\ M_{44} = |\mathbf{E}|^2 + |\mathbf{H}|^2 - k_0^2\left(|\mathbf{A}|^2 + |V|^2\right) \tag{6.53}$$

We recognise the maxwellian form, up to the mass terms, and we find :

[10] The factor μ_0 is surprising but according to (6,20) it disappears from the fields and potentials.

$$\int M_{\mu\nu}\, d\tau = \int T_{\mu\nu}\, d\tau \qquad (6.54)$$

§6.3) The photon spin.

Let us express the angular momentum with the nonsymmetric tensor $T_{\mu\nu}$:

$$m_{ik} = -\frac{i}{c}\int\left[x_i T_{4k} - x_k T_{4i}\right] d\tau \quad \left(i,k = 1,2,3\right) \qquad (6.55)$$

m_{ik} is *not* a constant of motion. But, like in Dirac's theory, we find a constant of motion m'_{ik} if we add a convenient term of spin :

$$m'_{ik} = m_{ik} + S_{ik} \qquad (6.56)$$

$$S_{ik} = i\hbar\int \phi^{+}\, \frac{b_4 a_i a_k + a_4 a_i b_k}{2}\, \phi \quad \left(i,k = 1,2,3\right) \qquad (6.57)$$

The dual $s_j = \varepsilon_{jik} S_{ik}$ of this tensor in $\mathbb{R}^3$ is a pseudo vector. In analogy with the Dirac spin, we find a *space-time pseudo-vector,* by adding a time component :

$$s_4 = c\hbar\int \phi^{+}\, \frac{b_4 a_1 a_2 a_3 + a_4 b_1 b_2 b_3}{2}\, \phi \qquad (6.58)$$

Now if we introduce in (6.55), the tensor : $T_{(\mu\nu)} = \frac{1}{2}\left(T_{\mu\nu} + T_{\nu\mu}\right)$, instead of the tensor : $T_{\mu\nu}$, we find the new momentum which is nothing but (6.57) :

$$m'_{ik} = -\frac{i}{c}\int\left[x_i T_{(4k)} - x_k T_{(4i)}\right] d\tau \quad \left(i,k = 1,2,3\right) \qquad (6.59)$$

Of course it is a conservative tensor. The difference with the theory of the electron, is that the eigenvalues of the matrices in the integrals (6.57) are: -1, 0, +1, instead of $\pm\frac{1}{2}$. We have a particle of maximum spin 1. The space-time pseudovector $s_\mu = \left\{\mathbf{s}, s_4\right\}$ has the following form in terms of electromagnetic quantities **in the case of the electric photon** :

$$\mathbf{s} = \frac{1}{c}\left[\mathbf{E}^* \times \mathbf{A} - \mathbf{A}^* \times \mathbf{E} + V^* \mathbf{H} + \mathbf{H}^* V\right]; \quad s_4 = \frac{1}{c}\left[\mathbf{A}^* . \mathbf{H} + \mathbf{H}^* . \mathbf{A}\right] \qquad (6.60)$$

Only terms corresponding to spin 1 appear in that formula : the terms corresponding to spin 0 vanish because $I_1 = 0$; it is not astonishing because $\mu_0 \neq 0$: see (6,22). If, we had started from (6,11) instead of (6,12), we should be nearer from Dirac's theory. Now consider the orbital momentum operator :

$$\mathbf{M}_{op} = \mathbf{r} \times \mathbf{p} \qquad (6.61)$$

This operator is not an integral of the motion but we can find a commutating operator by adding to M_{op} the new spin operators :

$$\mathbf{S} = \left\{ -i\hbar\left(\frac{a_2a_3 + b_2b_3}{2}\right), -i\hbar\left(\frac{a_3a_1 + b_3b_1}{2}\right), -i\hbar\left(\frac{a_1a_2 + b_1b_2}{2}\right) \right\} \tag{6.62}$$

which must be completed by:

$$S_4 = -\frac{i\hbar}{2}\left(a_1a_2a_3 + b_1b_2b_3\right) \tag{6.63}$$

which gives with **S** a relativistic quadrivector. The space components of **S** satisfy the spin commutation relations and finally these definitions will be used in the generalized theory of fusion.

§7) Relativistic noninvariance of the decomposition spin 1 - spin 0

The spin operators $s_j = \varepsilon_{jik} S_{ik}$ satisfy the commutation rules of an angular momentum and they have the eigenvalues : $\{-1, 0, 1\}$. The total spin $\mathbf{s}^2$ has the eigenvalues : $l(l+1) = (2,0)$, corresponding to $l = 1,0$.

In the case of a plane wave in (6,21), (6,22), and (6,29), (6,24), one can show that the group of equations (M) is associated with $l = 1$, with projections $s = -1,0,+1$ on the direction of propagation of the wave: $s = -1 \Leftrightarrow$ *right circular wave*, s=+1 $\Leftrightarrow$ *left circular wave* . For $s = 0$, we have in both cases a small longitudinal *electric* wave (due to the mass) for the electric photon, and a small longitudinal *magnetic* wave for the magnetic photon. The group (NM) is associated with $l = 0$.

So we can speak of (M) as a "spin 1 particle" and of (NM) as a "spin 0 particle". However, de Broglie made an important remark (*Broglie 9, Ch. VIII*): **although the equations (M) and (NM) are relativistically invariant, the separation between them is not covariant because it is based on the eigenvalues of the total spin-operator** $\mathbf{s}^2 = s_1^2 + s_2^2 + s_3^2$ **which is not a relativistic invariant**. The correspondence between the fieldvalues and the eigenvalues of $\mathbf{s}^2$ is :

1) For the electric photon:

$$\begin{matrix} \mathbf{A} & V & \mathbf{E} & \mathbf{H} & & I_1 & \mathbf{G} & W & I_2 & \mathbf{E'} & \mathbf{H'} \\ 2 & 0 & 2 & 2 & & 0 & 2 & 0 & 0 & 2 & 2 \end{matrix} \tag{6.64}$$

2) For the magnetic photon:

$$\begin{matrix} \mathbf{G'} & W' & \mathbf{H'} & \mathbf{E'} & & I_2 & \mathbf{A'} & V' & I_1 & \mathbf{H} & \mathbf{E} \\ 2 & 0 & 2 & 2 & & 0 & 2 & 0 & 0 & 2 & 2 \end{matrix} \tag{6.65}$$

In both cases, the first group corresponds to the (M) equations and the second group to (NM). We can note, when passing from (6.64) to (6.65), the following exchanges :

- Between potentials **A**, V and pseudopotentials $\mathbf{G}'$, W' ;
- Between fields **E**, **H** and anti-fields **E'**, **H'** [we know that **E'**, **H'**=0 in (6.64) and **E**, **H**=0 in (6.65)].
- Between I_1 and I_2, in the group (NM) ($I_1=0$ in (6.64) and $I_2=0$ in (6.65).

The most important fact is, that there are in both groups (M) and (NM), field quantities with $\mathbf{s}^2 = 2$ and $\mathbf{s}^2 = 0$ and thus spin 1 and spin 0 components : **there is no true separation between the values of spin.** De Broglie has shown (for both photons) that the separation only occurs in the proper system :

a) Because, for the electric photon, the potential $(\mathbf{A}, V)$ is *spacelike*, and the pseudopotential $(\mathbf{G}, W)$ is *timelike*, so that V and $\mathbf{G}$ disappear from (6.51), and only $\mathbf{s}^2 = 2$ remains in (M) ; conversely only $\mathbf{s}^2 = 0$ remains in (NM) because we know that : $\mathbf{E'} = \mathbf{H'} = 0$.

b) For the magnetic photon, the same happens, because this case follows from the preceding by multiplying an electric solution by γ_5, exchanging polar and axial quantities :

$$(\mathbf{E},\mathbf{H}) \leftrightarrow (\mathbf{H}',\mathbf{E}');\ (V,\mathbf{A}) \leftrightarrow (W,\mathbf{G});\ (I_1,I_2) \leftrightarrow (I_2,I_1) \qquad (6.66)$$

So that the potential $(\mathbf{A}, V)$ becomes *timelike* and the pseudopotential (**B**, W) becomes *spacelike*. And we have once more in the proper frame $\mathbf{s}^2 = 2$ in (M) and $\mathbf{s}^2 = 0$ in (NM), taking into account that we have $\mathbf{E} = \mathbf{H} = \mathbf{0}$ instead of $\mathbf{E'} = \mathbf{H'} = \mathbf{0}$.

In conclusion, the (M) and (NM) groups of equations cannot be rigorously separated, except in the proper frame, and they must be considered as forming one block, for two reasons :

1) The difficulty to separate spin 1 and spin 0 means finally that the composite photon cannot be considered as a spin 1 particle, but as a particle with a *maximum spin 1*, just as a two-electron atom or a two-atom molecule. It is noteworthy that the proper state in which the 1-components and 0-components are separated, is obviously the same for both components.

2) On the contrary, the presence of two photons (electric and magnetic) is inscribed in the very structure of the theory, their separation is covariant and is more radical than the separation of spin-states. The simultaneous presence, in (M) and (NM) equations, of potentials and pseudopotentials, of fields and anti-fields (even if half of them equal zero) and the "migration" of these quantities from one group of equations to the other, according to the type of photon, all these points constitute another link.

Of course, at the present stage of the problem, a question remains unsolved : what is, physically speaking, this spin 0 component? It could seem that all these questions are raised by the hypothesis : $\mu_0 \neq 0$. Of course, they could be avoided, admitting that : $\mu_0 = 0$. But it would be certainly a bad idea to shield the theory from a physical difficulty by a formal condition, at the expense of a more synthetic structure, as was shown above. A better answer will be given later, by the simple fact that the spin 0 component is a photon state which plays a physical role, just as the spin 1 state, and they must be included in the same global theory of light.

§8) The problem of a massive photon

We have seen that many features of de Broglie's theory of the photon including its logical coherence are due to the hypothesis : $\mu_0 \neq 0$. But, even if μ_0 is small, it implies many differences with the ordinary electromagnetism. These differences were examined in : (*Broglie 7,8,9* ; *Costa de Beauregard 2,3 ; Borne, Lochak, Stumpf ; Lochak 10, 18-21*).

§ 8.1) Gauge invariance. Obviously the common phase invariance disappears if $\mu_0 \neq 0$, which needs some comments :

a) First of all why do we find, in de Broglie's theory of light, the *Lorentz gauge* as a field equation ? Simply because it is the only relativistically invariant, linear differential law of the first order : it was the only possible.

b) Some practical problems. The relations between potentials and fields show that they are of the same order of magnitude. The mass terms are thus of k_0 order : very small. Therefore, in general, the gauge symmetry remains, up to a negligible error, and we can still chose with a good approximation the convenient gauge for most practical problems, provided that physics does not impose a particular choice.

c) In the present theory, the potentials are deducible from the fields, thus from observable phenomena : **they are no more mathematical fictions, but physical quantities**. It must be noticed that such a conception was already the one of Maxwell himself (see : *Maxwell*).

This is important for zero-field phenomena, only due to a potential, as the Aharonov-Bohm effect. The fact that the last effect is not gauge invariant is not an objection, because we know other physical quantities that are only partially defined by some effects but exactly defined by others : for instance, energy is defined by spectral laws, up to an additive constant, but exactly fixed by relativistic effects.

De Broglie gave another example of a physically defined potential : the electron gun (*Broglie 9*), in which the potential V between the electrodes is exactly defined for several reasons: 1) The *measurable* velocity of the emerging electron is given by the increase of energy, which is equal to eV. 2) The phase of the wave associated to the electron is relativistically invariant **only if** the frequency and the phase velocity obey the classical de Broglie formula, which imposes the gauge of V (the same as above). 3) The fundamental reason is that the intertia of energy does not allow an arbitrary choice of the origin of electrostatic potentials, which actually are not gauge invariant. They are physical quantities, related to mesurable effects. More recently Costa de Beauregard and Lochak published many other impressive experimental examples, in favor of the physical sense of electromagnetic potentials (*see references quoted above*).

d) A remark on the neutrino. After some attempts, de Broglie and other authors supposed that the Dirac particles that consitute by fusion photons and gravitons were *neutrinos*. For a long time, neutrino was considered as a massless particle, with arguments based on gauge invariance, separation of chiral - components, etc. But ideas changed : new theoretical arguments based on hypothetical oscillations between different kinds of neutrinos, the subsequent need of coupling constants, and some experimental evidences tend to a possible neutrino mass. If it is confirmed by facts, de Broglie's fusion theory will have as a consequence the prediction of a photon and a graviton mass, which will become in turn a credible idea. It must be confessed that the leptonic monopole theory (which is due to the author of these lines, who is a member of the same theoretical school) objectively disagrees with the last opinion. Sorry ! Nevertheless, it must be remembered (Ch 4 above) that we have also a theory of massive magnetic monopoles with the same symmetries, but it is a non linear theory, different from the present one.

§8.2) Vacuum dispersion.

If $\mu_0 \neq 0$, we can write :

$$h\nu = \frac{\mu_0}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} \rightarrow \mathrm{v} = c\sqrt{1-\frac{{\mu_0}^2 c^2}{h^2\nu^2}} \qquad (\mathrm{v} = \text{group velocity}) \qquad (6.67)$$

Thus, the vacuum must be dispersive, which was not yet observed but it may be stressed that the supposed value $\mu_0 < 10^{-45}\,g$ implies a Compton wavelength : $\lambda_c > 10^8\,cm = 10^3\,km$, so that the substitution of the Coulomb potential $\frac{1}{r}$ by the corresponding Yukawa potential $\frac{e^{-k_0 r}}{r}$ has a very small practical incidence, as on other numerical quantities. But the consequences on the symmetry laws are important.

Another question is that one could in principle observe a photon with a velocity smaller than c in the vacuum. At de Broglie's time, his estimations proved that it was impossible if $\mu_0 < 10^{-45}\,g$ (*Broglie 8,9).*

Nevertheless, with the progress of experimental physics, such a possibility must be reexamined and perhaps it may be considered rather as a question than as an objection.

§8.3) Relativity.

Practically, the velocity predicted for the photon is so near from c, that the difference has not any consequence (at least at the present level of knowledge). But the problem is : how shall we built the theory of relativity ? De Broglie's answer was one of his favorite jokes: "Light is not obliged to go with the velocity of light." In other words : we need, in relativity, a maximum invariant velocity but we do not need that this velocity is the velocity of light. It only happens that, in the vacuum, the velocity of light is near from it.

§8.4) Blackbody radiation.

In a given unit-volume there are $dn_\nu = \frac{4\pi\nu^2}{c^3}d\nu$ stationary waves of light in an elementary interval of frequencies, and we must have twice this number because of the transversality of light waves, which gives a factor 8 in the Planck law of blackbody radiation. But if $\mu_0 \neq 0$, it seems that we must multiply by 3 (instead of 2) because there is a longitudinal electric-component which gives 12 in the Planck law.

But this is wrong.The answer is the following : if we apply the formula for energy, it is shown that the longitudinal part of the field (so as the one, corresponding to potentials) is of the order of k_0, i.e. negligible (*Broglie 8,9*) so that it takes no part in the observed equilibrium and the factor 8 is the good one. This argument, given by de Broglie, was later independently confirmed by (*Bass and Schrödinger*).

§8.5) A remark on structural stability.

A physical theory has (at least) three truth-criteria : **experiment, logical consistency and structural stability**. The first two points are evident, the third is less. It means that a theory must have a sufficiant adaptability, to resist to slight experimental deviations, without destructing its mathematical frame.

Actually, most physical theories are too rigid and have structural *unstabilities* : for instance hamiltonian dynamics is structuraly unstable because its formalism doesn't allow the slightest dissipation. This means that the condition of structural stability, despite the strength of the argument and the high authority of the signatures, cannot be respected by all theories. But, at least, **one must eliminate arithmetical conditions or too precise symmetries,** which could not be verified experimentally.

An example is the mass of the photon. It is proved experimentally that the mass is *small,* but it cannot be proved that this mass is *exactly zero*, because it would be an *arithmetical condition*. In other words, electromagnetic gauge invariance – as a law of symmetry - may be proved approximately, not exactly.

It would be extremely worrying if electromagnetism needed exactly zero mass and gauge invariance [11]. And it is not so, but in virtue of Broglie's theory of photon, the smallness μ_0 implies negligible deviations in the experimental facts.

PART 2 : THEORY OF PARTICLES WITH MAXIMUM SPIN n

§9) Generalisation of the preceding theory.

[11] A theory of A. Eddington was based on 16 degrees of freedom and needed the *exact formula* $\frac{1}{\alpha} = \frac{16(16+1)}{2} + 1 = 137$ (α=fine structure constant). Unfortunately, the measure gives : $\frac{1}{\alpha}$=137,036...

The general theory is the subject of the second part of de Broglie's reference : (*Broglie 9*). We give only a short survey, even shorter than for the case of spin 1. The link with monopole will appear later.

§9.1) Generalized method of fusion.

Extending (6,7), the fusion of n Dirac equations gives a generalisation of the equations (6,8):

$$\frac{1}{c}\frac{\partial\phi_{ikl\ldots}}{\partial t} = a_k^{(p)}\frac{\partial\phi_{ikl\ldots}}{\partial x_k} + i\frac{\mu_0 c}{\hbar}a_4^{(p)}\phi_{ikl\ldots} \qquad \left(p = 1,2,\ldots,n\right) \tag{6.68}$$

Thus, we have *n* equations instead of *2*, and a 4^n component wave function (a spinor of *n-th* rank) instead of 16 components for the photon. And there are *4n* matrices $(a_r^{(p)})$ with 4^{2n} elements :

$$(a_r^{(p)})_{ik\ldots opq\ldots,i'k',\ldots o'p'q'\ldots}=\delta_{ii'}\delta_{kk'}\ldots\delta_{oo'}(\alpha_r)_{pp'}\delta_{qq'}\ldots \tag{6.69}$$

They obey the (6,10) relations:

$$a_r^{(p)}a_s^{(p)}+a_s^{(p)}a_r^{(p)}=2\delta_{rs};\ a_r^{(p)}a_s^{(q)}-a_s^{(q)}a_r^{(p)}=0 \quad (\text{if } p\neq q) \tag{6.70}$$

The same problem as in equations (6,8), occurs here: there are n times too much equations (for the photon, we had twice). We have indeed $n4^n$ equations for 4^n components of the wave function. The answer is almost the same.

§9.2) "Quasi-Maxwellian" form.

We shall proceed as in §**3.1).** But we put, at first:

$$F^{(p)} = a_k^{(p)}\frac{\partial}{\partial x_k} + i\frac{\mu_0 c}{\hbar}a_4^{(p)} \tag{6.71}$$

We have the relations:

$$F^{(p)}F^{(q)} = F^{(q)}F^{(p)}, \quad \forall p,q; \quad \left(F^{(p)}\right)^2 = \Delta - k_0^2 \tag{6.72}$$

which implies that the wave obeys the Klein-Gordon equation. Now, (6.68) takes the form :

$$\frac{1}{c}\frac{\partial\phi}{\partial t} = F^{(p)}\phi;\ p = 1,2,\ldots n \tag{6.73}$$

By adding these equations, we find a new evolution equation generalising the equation (A) in (6,11) :

$$(\mathrm{A})\ \frac{1}{c}\frac{\partial\phi}{\partial t} = F\phi; \quad F = \frac{1}{n}\sum_{p=1}^{n}F^{(p)} \tag{6.74}$$

Now, substracting the equations (6.73) one from each other in a convenient way, we can eliminate the time derivatives and find (n-1) "condition equations". It may be done in many ways. For instance we can choose the following system, similar to the equation (B) in (6,11) :

(B) $G^{(p)}\phi = \dfrac{F^{(1)} - F^{(p)}}{2}\phi = 0 \quad \left(p = 2,3,\ldots,n\right)$ (6.75)

It is easy to prove that the new system (A), (B) is equivalent to (6.68) or (6.72). Owing to (6.69), one can see that F and G commute, but their product doesn't equal zero, contrary to what happened with the operators of the second members of (6,11) in the special case $n = 2$:

$$G^{(p)}F = FG^{(p)} \neq 0 \qquad (6.76)$$

This means that, contrary to (6,11), we cannot prove, using (6.74) and (6.75), that the conditions (B) are deducible from the evolution equation (A). However, as a consequence of (6.63), the first members $G^{(p)}\Phi$ of (6.74) are solutions of (6.73), so that, if the conditions (B) are satisfied at an initial time $t = 0$, they are satisfied at every time.

On the other side, we can prove the compatibility of the *(n-1)* equations (B), so that the compatibility of the system (6.67) - or, equivalently of (6.73), (6.74) - is proved.

§9.3) The density of quadri - current.

Generalising the case of maximum spin 1 de Broglie introduced another set of matrices (*Broglie 9*) :

$$B_4^{(p)} = a_4^{(1)}a_4^{(2)}\ldots a_4^{(p-1)}a_4^{(p+1)}\ldots a_4^{(n)}, \qquad (6.77)$$

Each is the product of all the $a_4^{(i)}$ except the one corresponding to the index *p.* The quadri-current density is the following and it is easy to verify that it is conservative :

$$J_k = -c\phi^* \frac{1}{n}\sum_{p=1}^{n} a_k^p B_4^p \phi; \quad \rho = \phi^* \frac{1}{n}\sum_{p=1}^{n} B_4^p \phi; \quad \frac{\partial \rho}{\partial t} + \partial_k J_k = 0 \qquad (6.78)$$

Generalising a remark made in § 5.2, it is interesting to examine the ρ *density*. Following de Broglie, we shall do it in the case of the plane wave. Let us note, by the way, that it is not difficult to calculate a plane wave for a particle of maximum *spin* $n/2$: the phase is evident and the amplitudes are given by the *n* products of *4* amplitudes of *n* Dirac plane waves, which gives 2^n constants restricted by the fusion conditions. The calculation is rather long (*Broglie 9*), but the result is simple. We find :

$$\overline{Q} = \rho Q$$

With : (6,79)

$$\rho = \left(\frac{\mu_0 c^2}{W}\right)^{n-1} |\phi|^2 \quad (\mu_0 \textit{ and } n = \textit{mass and number of spin } 1/2 \textit{ particles}) \qquad (6.80)$$

We see that:

- If *n* is odd, the sign of ρ is definite positive as in the case *n=1* of a Dirac electron.
- If *n* is even, ρ has the same sign as energy, it is indefinite : it was the case of the photon (*spin 1*) and it is the case for a graviton (*spin 2*).

It is interesting to note, with de Broglie, the curious presence, in (6.80), of the (n-1)th power of the Lorentz contraction, which means that the density ρ, integrated on a volume ($\int \rho d\mathrm{v}$), will be contracted exactly *n* times (the number of elementary *spin 1/2* particles). The exception is the Dirac particle, for which : *n-1=0*, so that the factor disappears and the integral is only contracted by the integration-volume itself. De Broglie conjectured that this factor is perhaps an echo of a hidden spatial structure of the composite particle that we can describe only as a point, in the present state of linear quantum mechanics.

§9.4) The energy density.

We begin with an elementary calculation of the *energy density,* using the preceding density ρ for a plane wave. The definition of the density ρ means that all the mean values are obtained by the integration of a physical quantity multiplied by ρ.

The energy density is thus obtained (in the case of a plane wave) owing to the formula (6.80) :

$$\rho W = \left(\frac{\mu_0 c^2}{W} \right)^{n-1} W |\phi|^2 \qquad (6.81)$$

Here, the power of *W* is not *(n-1)* but *(n-2),* so that we find a result opposite to the result for ρ:

- If *n* is odd $\rho\, W$ has the same indefinite sign as energy : it was the case for n=1, for the Dirac electron.
- If *n* is even, the sign of $\rho\, W$ is definite-positive, as it was for the photon and as it will be for the graviton. This is confirmed by more sophisticated calculations using the energy tensor density.

We shall introduce two classes of tensors. The first, named "corpuscular" by de Broglie, is given by the receipts of quantum mechanics. The second class, called by de Broglie "of type M" (M for Maxwell), is more wide and is inspired by electromagnetism.

§9.5. The "corpuscular" tensor.

We make use of the B matrices definited in (6.77) with the following notations:

$$\rho\, U_i^{(p)} = a_i^{(p)} \quad (i = 1,2,3,), \quad U_i^{(p)} = 1; \quad (p = 1,2,...,n) \qquad (6.82)$$

The tensor is then (*Broglie 9*), generalising the spin 1 case :

$$T_{\mu\nu} = T_{\nu\mu} = \frac{\hbar c}{4in} \sum_{p=1}^{n} \begin{bmatrix} \phi^* U_\mu^{(p)} B_4^{(p)} \dfrac{\partial \phi}{\partial x_\nu} - \dfrac{\partial \phi^*}{\partial x_\nu} U_\mu^{(p)} B_4^{(p)} \phi \\ +\phi^* U_\nu^{(p)} B_4^{(p)} \dfrac{\partial \phi}{\partial x_\mu} - \dfrac{\partial \phi^*}{\partial x_\mu} U_\nu^{(p)} B_4^{(p)} \phi \end{bmatrix} \qquad (6.83)$$

We verify its conservation in virtue of the equations:

$$\partial_\nu T_{\mu\nu} = \partial_\mu T_{\mu\nu} = 0 \qquad (6.84)$$

It is interesting to verify that the tensor takes, for a plane wave, the form that is to be expected and we find indeed the following matrix for its components (**p**=momentum, **v**=group velocity) :

$$\begin{Bmatrix} \rho p_1 v_1 & \rho p_1 v_2 & \rho p_1 v_3 & \rho p_1 c \\ \rho p_2 v_1 & \rho p_2 v_2 & \rho p_2 v_3 & \rho p_2 c \\ \rho p_3 v_1 & \rho p_3 v_2 & \rho p_3 v_3 & \rho p_3 c \\ \rho p_1 c & \rho p_2 c & \rho p_3 c & \rho W \end{Bmatrix} \tag{6.85}$$

In particular T_{44} is the quantity (6.81).

§9.6) The "type M" tensors.

At first, we shall generalize the formula (6.77) by the definition of a set of operators of rank m :

$$B_4^{(pq\ldots)} = a_4^1 a_4^2 \ldots a_4^{p-1} a_4^{p+1} \ldots a_4^{q-1} a_4^{q+1} \ldots \tag{6.86}$$

It is the product of all the a_4^r *(r=1, 2,...,n),* excepting those for which *r* is equal to one of the *m* indices *p, q* ... of *B*. Using of these operators and of (6.81), we define a set of tensors of rank *m* (*Broglie 7)*.

$$M_m = \mu_0 c^2 \phi^* \frac{\sum_{pq\ldots} U_i^{(p)} U_j^{(q)} \ldots B_4^{(pq\ldots)}}{a_n^m} \phi; \quad \left(a_n^m = \frac{n!}{(n-M)!} \right) \tag{6.87}$$

These tensors are obviously symmetric, but we keep only those, the rank *m=2r* of which is even. Thus we have defined (for a particle of maximum spin *n*) *n/2* tensors if *n* is even and *(n-1)/2* tensors if *n* is odd. Finally, we contract each tensor of rank *2r*, over *2r-2* indices, which gives a number equal to the half of the greatest even number contained in *n* of tensors of rank two, according to the formula :

$$M_{ij}^{(r)} = \sum_{ijkl\ldots}^{4} M_{ijkl\ldots}^{kl\ldots} \tag{6.88}$$

And we must remember that, applying the receipt to real space-cordinates, we must change the sign when indices 1,2,3 go up or down.

These tensors were defined by de Broglie as tensors "of type M". In virtue of the general equations (6.87), we have, just as for the tensor T :

$$\partial_\nu M_{\mu\nu}^{(r)} = \partial_\mu M_{\mu\nu}^{(r)} \tag{6.89}$$

and we have *n*/2 tensors $M^{(r)}$ of rank 2 if *n* is even and (*n-2*)/2 tensors if *n* is odd.

A priori, each conservative tensor may be considered as an impulse-energy tensor and it may be shown that, *for a plane wave*, ∀ r every tensor $M_{\mu\nu}^{(r)}$ gives exactly the table of components (6.83). It is not true for other solutions, but it remains true integrally :

$$\int T_{\mu\nu} d\tau = \int M_{\mu\nu}^{r} d\tau; \quad \forall r \tag{6.90}$$

§9.7) Spin

Starting from the formula (6.72) - generalisation of (6,11) - we have the same orbital operator, and the spin operators are now :

$$S_i = \hbar \sum_{p=1}^{n} s_i^{(p)}; \quad (i = 1,2,3); \quad S_4 = \hbar \sum_{p=1}^{n} s_4^{(p)} \tag{6.91}$$

It would be difficult to reproduce here the general nomenclature of spin states and (for an even number of spin 1/2 particles) the decomposition of wave functions in terms of tensor components. This nomenclature is based on the Clebsch-Gordan theorem for the product of irreducible representations, but it is completed in (*Broglie 7*)*,* who defines the set of independent constants of a plane wave and the symmetry of tensors defined by an even number of particles.

These problems are treated in a different form, in (*Fierz 1*), the work of which is based not on the fusion theory but on some conditions added to the field obeying the Klein-Gordon equation, to describe a spin n/2 particle. This point of view was developed in (*Fierz & Pauli 2, 3*) and on the basis of a previous work of Dirac on the generalisation of the equation of the electron, for higher spin-values (*Dirac 2*).

PART 3 : THEORY OF PARTICLES WITH MAXIMUM SPIN 2

§10) The particles of maximum spin 2. Graviton.

(*Fierz & Pauli 2*) were the first who found the analogy between the equation of a *particle of spin 2* and the linear approximation of the Einstein equation of a gravitation-field. This approximation was given by (*Einstein 2, 3*) himself. It may be found for instance in (*Laue,* or *Möller*). The paper (*Einstein 2*) was the first in which Einstein formulated the idea of gravitational waves. He even alluded to a possible modification of gravitation theory by quantum effects, in analogy with the modification of Maxwell's electromagnetism.

It must be stressed that the quantum theory of gravitation, developed by L. de Broglie and M.-A. Tonnelat (*Broglie 9*, *Tonnelat1,2*) on the basis of the fusion method, is not based on a particle of spin 2 but on *the particle of maximum spin 2*. This is an important point for two reasons :

1) The fusion theory raises the question : is the graviton a *composite particle*, just as the photon and all particles of spin higher than ½ ?

2) In the last theory gravitons don't appear alone. They are linked to photons. **This theory is actually a unitary theory** of gravitation and electromagnetism (at least at the linear approximation) and the fields are not gathered by an extended geometry, but by the fusion of spins.

§10.1) Why gravitation and electromagnetism are linked ?

Formally one could say that "Fields are linked by Clebsch-Gordan's theorem" because :

$$D_{\frac{1}{2}} \times D_{\frac{1}{2}} \times D_{\frac{1}{2}} \times D_{\frac{1}{2}} = D_2 + 3D_1 + 2D_0 \tag{6.92}$$

Therefore, in the fusion of four spin 1/2 particles, we must find : one particle of spin 2, three particles of spin 1 and two particles of spin 0. In particular, we have gravitons and photons,

To which we must add the spin 0 photons, the physical meaning of which is related to the Aharonov-Bohm effect, as it was developped in the Part 1 of §4 ss.

De Broglie gave an interesting argument : he defined a particle of maximum spin 2 by the fusion of two particles of spin 1, described by quadripotentials : $A_\mu^{(1)} = \left\{ A^{(1)}, V \right\}$, $A_\mu^{(2)} = \left\{ A^{(2)}, V \right\}$, and the invariants $I_2^{(1)}, I_2^{(2)}$ ($I_1^{(1)}, I_1^{(2)} = 0$ because $\mu_0 \neq 0$ and we consider only the electric case). The fusion gives:

$$A_\mu^{(1)} \times A_\mu^{(2)};\ A_\mu^{(1)} \times I_2^{(2)};\ I_2^{(1)} \times A_\mu^{(2)};\ I_2^{(1)} \times I_2^{(2)} \tag{6.93}$$

The first product is a tensor of rank 2 that defines a symmetric and an antisymmetric tensor:

$$A_{(\mu\nu)} = \frac{A_{(\mu\nu)} + A_{(\nu\mu)}}{2};\ A_{[\mu\nu]} = \frac{A_{(\mu\nu)} - A_{(\nu\mu)}}{2} \tag{6.94}$$

The products $A_\mu^{(1)} \times I_2^{(2)}$ and $I_2^{(1)} \times A_\mu^{(2)}$ are vector-like quantities $P_\mu^{(1)}$, $P_\mu^{(2)}$ and it may be hoped that they will be photon potentials. The antisymmetric tensor $A_{[\mu\nu]}$ suggests the electromagnetic-field.

The symmetric tensor $A_{(\mu\nu)}$ cannot be interpreted at this level of exposition, but actually we can guess in advance that it will be related to gravitation.

De Broglie shows, owing to a study of plane waves, that $P_\mu^{(1)}, P_\mu^{(2)}$ and the antisymmetric tensor $A_{[\mu\nu]}$ are related to the spin 1; $A_{(\mu\nu)}$ is linked to spin 2, only if it is reduced to a zero-spur tensor because $sp\,A_{(\mu\nu)} = A_{(\mu\mu)}$ is an invariant and it will be actually related to spin 0, just as the invariant $I_2^{(1)} \times I_2^{(2)}$.

Now it must be remembered that, as it was shown in the case of the photon, *the splitting between different spin-states is not relativistically covariant* because it is based on the total spin operator which is not a relativistic invariant. **Therefore, in the fusion theory, gravitation cannot appear without electromagnetism.** Furthermore, it will be shown that, if $\mu_0 \neq 0$, the splitting between spin 2 and spin 0 is impossible, and the interpretation of this fact is highly interesting.

§10.2) The tensorial equations of a particle of maximum spin 2.

We give only the tensorial form generalising § 4.1. The total wave equations (type (6,11) for $n = 4$) would have 4^4=256 components with 168 independent quantities (*Broglie 7*) :

(A)

$$\begin{aligned} \partial_\mu \phi_{(\nu\rho)} - \partial_\nu \phi_{(\mu\rho)} &= k_0 \phi_{[\mu\nu]\rho} \\ \partial_\rho \phi_{[\rho\mu]\nu} &= k_0 \phi_{(\mu\nu)} \\ \partial_\mu \phi_{[\rho\sigma]\nu} - \partial_\nu \phi_{[\rho\sigma]\mu} &= k_0 \phi_{[\mu\nu][\rho\sigma]} \\ \partial_\varepsilon \phi_{([\varepsilon\rho][\mu\nu])} &= k_0 \phi_{[\mu\nu]\rho} \end{aligned} \tag{6.95}$$

$\phi_{(\mu\nu)}$ is a symmetric tensor of rank 2, $\phi_{[\mu\nu]\rho}$ a tensor of rank 3 antisymmetric with respect to the two first indices, $\phi_{[\mu\nu][\rho\sigma]}$ a tensor of rank 4 antisymmetric with respect to $\mu\nu$ and $\rho\sigma$, but symmetric with respect to these pairs. A consequence of (6.95) is :

$$\begin{aligned} &\partial_\nu \phi_{(\mu\nu)} = \partial_\rho \partial_\nu \phi_{[\rho\mu]\nu} = 0 \\ &\phi_{[\rho\rho]} = \frac{1}{2} \phi_{[\mu\rho][\mu\rho]};\quad \partial_\nu \phi_{(\rho\rho)} = k_0 \phi_{[\nu\rho]\rho} \end{aligned} \tag{6.96}$$

The group (B) is divided in three sub-groups where appear new tensors of rank 2, 3, 4:

(B1)

$$
\begin{aligned}
&\partial_\mu \phi^{(1)}_{(\nu\rho)} - \partial_\nu \phi^{(1)}_{(\mu\rho)} = k_0 \phi^{(1)}_{[\mu\nu]\rho} \\
&\frac{1}{2}\left(\partial_\rho \phi^{(1)}_{[\rho\mu]\nu} - \partial_\rho \phi^{(1)}_{[\rho\nu]\mu}\right) = k_0 \phi^{(1)}_{[\mu\nu]} \\
&\partial_\mu \phi^{(1)}_{[\rho\sigma]\nu} - \partial_\nu \phi^{(1)}_{[\rho\sigma]\mu} = k_0 \phi^{(1)}_{[\mu\nu][\rho\sigma]} \\
&\partial_\varepsilon \phi^{(1)}_{([\varepsilon\rho][\mu\nu])} = k_0 \phi^{(1)}_{[\mu\nu]\rho}
\end{aligned}
\tag{6.97}
$$

Note the antisymmetries (square brakets). From (6.97) we deduce the identities :

$$\phi^{(1)}_{[\nu\mu]\nu} = \phi^{(1)}_{([\mu\nu][\rho\nu])} = 0 \tag{6.98}$$

The equations (B2) and (B3) are identic and we have :

(B2, B3)

$$
\begin{aligned}
&\partial_\mu \chi^{(1)}_\nu - \partial_\nu \chi^{(1)}_\mu = k_0 \chi^{(1)}_{[\mu\nu]} \\
&\partial_\rho \chi^{(1)}_{[\rho\nu]} = k_0 \chi^{(1)}_\nu \\
&\partial_\mu \chi^{(1)}_\nu = k_0 \chi^{(1)}_{\rho\nu} \\
&\partial_\rho \chi^{(1)}_{[\mu\nu]} = k_0 \chi^{(1)}_{[\mu\nu]\rho}
\end{aligned}
\tag{6.99}
$$

In the third equation $\chi^{(1)}_{\rho\nu}$ is neither symmetric nor antisymmetric. (6.99) entails :

$$
\begin{aligned}
&\chi^{(1)}_{\rho\rho} = 0;\ \chi^{(1)}_{\mu\nu} - \chi^{(1)}_{\nu\mu} = \chi^{(1)}_{[\mu\nu]} \\
&\chi^{(1)}_{[\nu\rho]\rho} = -\chi^{(1)}_\nu;\ \chi^{(1)}_{[\mu\nu]\rho} + \chi^{(1)}_{[\nu\rho]\mu} + \chi^{(1)}_{[\rho\mu]\nu} = 0
\end{aligned}
\tag{6.100}
$$

Finally we find a last group of equations:

(C)

$$
\begin{aligned}
&\partial_\mu \phi^{(0)}_\nu = \partial_\nu \phi^{(0)}_\mu = k_0 \phi^{(0)}_{(\mu\nu)} \\
&\partial_\mu \phi^{(0)}_\mu = k_0 \partial_\mu \phi^{(0)} \\
&\partial_\mu \phi^{(0)} = k_0 \phi^{(0)}_\mu
\end{aligned}
$$

6.101)

The equations (B1), (B2), (B3) are three realisations of total spin 1. It is evident for (B2), (B3) because putting :

$$F_\mu = k_0 \chi^{(1)}_\mu;\ F_{[\mu\nu]} = k_0 \chi^{(1)}_{\mu\nu} \tag{6.102}$$

and defining potentials and fields as we did in (6,20), we find the Maxwell equations with mass (we shall see that it needs some comments).

The correspondence is less evident for (6.97). Instead of (6.102), we must write :

$$F_\mu = \frac{k_0}{6} \varepsilon_{\mu\lambda\nu\rho} \phi^{(1)}_{[\lambda\nu]\rho};\ F_{[\mu\nu]} = k_0 \phi^{(1)}_{\mu\nu} \tag{6.103}$$

($\varepsilon_{\mu\lambda\nu\rho}$ = Levi-Civita symbol). Applying (6,20), we find the Maxwell equations.

Now, (C) is a realisation of spin 0 as it may be seen by comparison of (6.101) with (6,22). But here we find a difficulty which justifies the preceding remarks : de Broglie (who didn't know the magnetic case), considered only the electric photon (6,21) and he identified (6.101) with the non maxwellian equations (6,22). But it implies the identity $\varphi^{(0)}=I_2$, where $\varphi^{(0)}$ is a *scalar* while I_2 is a **pseudoscalar**.

In the time of the reference : 1943 *(Broglie 7*), people was less careful than now, with parity and de Broglie wrote that (6.101) and (6,22) **"are entirely equivalent (at least when vectors and pseudo-vectors are assimilated)".** In our days, we pay more attention to parity and we cannot neglect such a discrepancy : **an equality like $\phi^{(0)} = I_2$ is unacceptable**. There are two possible solutions :

1) We could admit that $\phi^{(0)} = I_2$, if $\phi^{(0)} = I_2 = 0$. Thus the spin 0-component (C) vanishes. But there is a second spin 0-component, hidden in the equations (A) in the form of an invariant $\phi^{(0)}$, a vector $\phi^{(0)}_\mu$ and a symmetric tensor $\phi^{(0)}_{(\mu\nu)}$, that we can define as:

$$\phi^{(0)} = \phi^{(0)}_{(\rho\rho)}\,;\ \phi^{(0)}_\mu = \phi^{(0)}_{[\mu\rho]\rho}\,;\ \phi^{(0)}_{(\mu\nu)} = \phi_{([\mu\rho][\nu\rho])} - \phi_{(\mu\nu)} \qquad (6.104)$$

One can show using (6.94) that these tensors obey the group C of equations (6.101), but once more, if $\phi^{(0)}$ is a true scalar, we can write $\phi^{(0)} = I_2$ only if $\phi^{(0)} = I_2 = 0$. It implies that (6. 101) is submitted to the condition $sp\,\phi^{(0)}_{(\rho\rho)} = 0$, that was *a priori* supposed by Fierz and Pauli, who based their theory on a spin 2 (and not maximum spin 2) particle. De Broglie criticized this postulate as artificial.

The above suggestion, based on parity, could be considered as the justification of their hypothesis. However it may be objected as it was shown by de Broglie, that the splitting of spin components is not covariant. It is, at least, the case for the condition : $\phi^{(0)} = I_2 = 0$, despite that the equality $sp\,\phi^{(0)}_{(\rho\rho)} = 0$ is covariant : the problem thus remains unsolved. But there is a second proposition :

2) We can ask the question: is $\phi^{(0)} = I_2$ the good equality? Perhaps it is rather $\phi^{(0)} = I_1$, which is covariant because I_1 is a true invariant. In such a case, (6.101) must not be identified with (6,22), but with (6,27). Is it possible ? It seems that yes.

Let us go back to (6.92). The products $A^{(1)}_\mu \times I^{(2)}_2$ and $I^{(1)}_2 \times A^{(2)}_\mu$, denoted : $P^{(1)}_\mu$, $P^{(2)}_\mu$, were considered by de Broglie as vectors, but he said, more prudently, "vector-like": **actually, they are pseudo-vectors, because they are the products of a polar-vector by a pseudo-scalar. Therefore $P^{(1)}_\mu$ and $P^{(2)}_\mu$ are not polar potentials but pseudo-potentials of magnetic type as those that appear in (6,26)**. On the contrary, the product $I^{(1)}_2 \times I^{(2)}_2$ of two pseudo-scalars is a *true* scalar, of the same type as I_1, which appears in (6,27) and they can be identified.

The answer to the difficulty is that the third photon associated to the graviton is not electric but magnetic.

Let us suppose that, instead of introducing only electric photons, we introduce a magnetic photon in the symbolic formulae (6.92) with pseudo-potentials $G^{(1)}_\mu$, $G^{(2)}_\mu$ and pseudoscalars $I^{(1)}_2, I^{(2)}_2$. The fusion gives:

$$G^{(1)}_\mu \times G^{(2)}_\mu;\quad G^{(1)}_\mu \times I^{(2)}_\mu;\quad I^{(1)}_\mu \times G^{(2)}_\mu;\quad I^{(1)}_\mu \times I^{(2)}_\mu \qquad (6.105)$$

and we see that:

- The spin 2 product : $G_\mu^{(1)} \times G_\mu^{(2)}$ has the same symmetry as $A_\mu^{(1)} \times A_\mu^{(2)}$, because the axial character of $G_\mu^{(1)}, G_\mu^{(2)}$ is annihilated by the product.

- For the same reason, the spin 0 product $I_1^{(1)} \times I_1^{(2)}$ is a scalar, as was $I_2^{(1)} \times I_2^{(2)}$.

- The spin 1 products $G_\mu^{(1)} \times I_\mu^{(2)}$; $I_\mu^{(1)} \times G_\mu^{(2)}$ are pseudo-vectors, as $A_\mu^{(1)} \times I_2^{(2)}$; $I_2^{(1)} \times A_\mu^{(2)}$: they are products of a pseudo-vector by a scalar, while the latter were products of a polar vector by a pseudo-scalar.

Thus we find a magnetic photon wether we start from electric or from magnetic photons and we can assert that one of the photons associated to the graviton is not electric but magnetic.

PART 4 : QUANTUM (LINEAR) THEORY GRAVITATION

§11) The particle of maximum spin 2. Graviton (see : *Broglie 9 and Tonnelat 3)* .

Now, we shall follow de Broglie and Tonnelat and consider the general equations (A) when $sp\,\phi_{(\rho\rho)}^{(0)} \neq 0$. But we shall not be able to separate the spin 2 component from its spin 0 part !

We start from (6.81), (6.82) and the Klein-Gordon équation, verified by all the field quantities :

$$\square\phi = -k_0^2\phi \quad \left(\square = -\partial_\rho\partial_\rho\right) \tag{6.106}$$

The metric tensor $g_{(\mu\nu)}$ will be taken at the linear approximation:

$$g_{(\mu\nu)} = \delta_{\mu\nu} + h_{(\mu\nu)} \quad \left(h_{(\mu\nu)} \ll 1\right) \tag{6.107}$$

At this limit, the propagation of gravitation waves is given by:

$$\square g_{(\mu\nu)} = -2R_{(\mu\nu)} \quad \left(R_{(\mu\nu)} = g^{\rho\sigma} R_{([\mu\rho][\nu\sigma])}\right) \tag{6.108}$$

Where $R_{([\mu\rho][\nu\sigma])}$ is the tensor of Riemann-Christoffel ; in the euclidian regions of space-time we have the d'Alembert equation $\square g_{(\mu\nu)} = 0$ without second member. This is true if we use "isothermic" coordinates x_μ, for which $D_2 x_\mu = 0$; D_2 is the second order Beltrami differential parameter.

Now it seems that metrics could be defined by:

$$g_{(\mu\nu)} = \phi_{(\mu\nu)} \tag{6.109}$$

But Tonnelat remarked that, according to (6.98), this implies: $\partial_\mu g_{(\mu\nu)} = 0$, which is wrong because "isothermic" coordinates obey the relation[12] :

[12] It must be noted that we have not: $g_\rho^\rho = g_{\rho\sigma} g^{\rho\sigma}$, because this quantity, *in the present case,* is equal to 4.

$$\partial_\mu g_{(\mu\nu)} = \frac{1}{2}\partial_\nu g_{(\rho\rho)} \quad \left(g_{(\rho\rho)} = g_{(\mu\nu)}\delta^{(\mu\nu)}\right) \tag{6.110}$$

And the second member is not equal to zero. (6.96) thus contradicts (6.95). This is why Tonnelat suggested the following metrics (which is possible because : $k_0 \neq 0$) :

$$g_{(\mu\nu)} = \phi_{([\mu\rho][\nu\sigma])} = \phi_{(\mu\nu)} + \frac{1}{k_0^2}\partial_\mu\partial_\nu\phi_{(\rho\rho)} \tag{6.111}$$

From which it follows immediately:

$$\partial_\mu g_{(\mu\nu)} = \partial_\mu\phi_{([\mu\rho][\nu\sigma])} = \partial_\nu\phi_{(\rho\rho)} \tag{6.112}$$

So we get from (6.82), (6.111) and (6.112) :

$$g_{(\rho\rho)} = 2\phi_{(\rho\rho)} \to \partial_\mu g_{(\mu\nu)} = \frac{1}{2}\partial_\nu g_{(\rho\rho)} \tag{6.113}$$

in accordance with ((6.96).

Now, from (6.97) we deduce that $g_{(\mu\nu)}$ obeys the Klein-Grordon equation, as other field-quantities:

$$\Box g_{(\mu\nu)} = -k_0^2 g_{(\mu\nu)} \tag{6.114}$$

We have to identify (6.114) with (6.94), so that :

$$R_{(\mu\nu)} = \frac{1}{2}k_0^2 g_{(\mu\nu)} \tag{6.115}$$

Now, the tensor of Riemann-Christoffel may be deduced at the linear approximation, from (6.111), (6.81) and (6.82) :

$$\phi_{([\mu\rho][\nu\sigma])} \cong \frac{2}{k_0^2}\, R_{([\mu\rho][\nu\sigma])} \tag{6.116}$$

This formula is possible only if : $\mu_0 \neq 0$, which imposes a curvature of the universe. Indeed, $\frac{k_0^2}{2}$ is nothing but the cosmological constant (unfortunately, Einstein disliked it !) defined by:

$$R_{(\mu\nu)} = \lambda g_{(\mu\nu)} \tag{6.117}$$

λ is related to a "natural curvature" of space-time. In the euclidian space: $\lambda = 0$; in a de Sitter space of radius R we have : $\lambda = \frac{3}{R^2}$. Therefore:

$$\lambda = \frac{k_0^2}{2} = \frac{\mu_0^2 c^2}{2\hbar^2} \tag{6.118}$$

And the graviton mass is related to a natural curvature of radius R :

$$\mu_0 = \frac{\hbar\sqrt{6}}{Rc} \qquad (6.119)$$

If $R = 10^{26}\, cm$, the graviton (and photon) mass is:

$$\mu_0 = 10^{-66}\, g \qquad (6.120)$$

The spin 0 may be eliminated from the equations of spin 2 only in two cases:
- either by the a priori supposition that $\Phi^{(0)}=0$ (Fierz equations),
- or *at the limit case* $\mu_0=0$, when the radius of the universe is infinite: the euclidian case[13].

In conclusion, the quantum theory of gravitation based on de Broglie's fusion theory raises the important question of a composite nature of photon and graviton and above all the theory furnishes the beginning of the quantum unitary field-theory of electromagnetism and gravitation. Only the beginning because it is linear.

Two remarks may be done :

- It could be asked if the obstinate efforts of Einstein and other great physicists and mathematicians towards a unitary field theory had any sense, given that we know hundreds of particles and it could seem that there is no reason to pay a particular attention to two of them : photon and graviton. De Broglie's theory gives a reason: these particles are the only that are linked by spin properties, in the fusion procedure. This argument is exterior to the geometrical path followed by Einstein.

- The second remark concerns symmetry : the fact that a photon associated to the graviton could be magnetic instead of electric, as was suggested above, signifies the intrusion of duality, chirality, magnetic monopoles instead of electric charges and so on. **It is certainly of interest that a photon is perhaps not the one that was expected and it must be stressed that there is another photon with a zero spin.**

§12) Some words about the comparison with other theories.

First of all, we must emphasize the priority of Louis de Broglie in the quantum theory of photon considered as a *composite particle.* His first paper appeared in 1934 (*The wave equation of the photon, Broglie 4*) and the idea of a *fusion* of Dirac particles is the starting point of his theory of particles of higher spin.

A second point is that, unlike the others, de Broglie's initial aim was not a generalisation of Dirac's equation but a theory of light. This is why he didn't introduce any electromagnetic interaction.

For reasons given above, he was the only to suppose a massive photon, contrary to other authors who considered a massless photon as an evidence. He never tried to extend his theory to massless particles and even scarcely alluded to this possibillity.

§12.1. The "Proca equation".

The equations (6,21) and the very idea of a massive photon are often ascribed to Proca. Actually, it is the result of a misunderstanding, if not a "misreading".

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[13] These problems are carefully examinated in : (*Broglie 7*).

1) The "Proca equations" (see *Proca*) appeared in 1936, two years after the Broglie equations *(Broglie 4*). Moreover, the paper of Proca was untitled : *"On the ondulatory theory of positive and negative electrons"*. **It was not a theory of the photon but a theory of the electron ! :** an attempt to avoid the negative energies, as it was frequent in that time[14].

2) Rejecting spinorial wave functions, Proca suggests – for the electron – a *vectorial equation* deriving from the Lagrangian:

$$L = \frac{\hbar^2 c^2}{2} G^*_{rs} G_{rs} + m_0^2 c^4 \psi^*_r \psi_r$$
$$G_{rs} = \left(\partial_r - iA_r\right)\psi_s - \left(\partial_s - iA_s\right)\psi_r \ \left(r, s = 1,2,3,4\right) \qquad (6.121)$$

The complex vectorial function ψ_r of the *electron* takes the place of de Broglie's *photon* potential $(\mathbf{A}, V)$; and A_r is a real potential of an *external* electromagnetic field acting on the electron ψ_r, and the elecron - not the photon - was the object of the theory ! From (6.121), Proca derived the equations :

$$\left(\partial_r - iA_r\right)G_{rs} = k^2\psi_s, \left(\partial_r + iA_r\right)G^*_{rs} = k^2\psi^*_s \ \left(k = \frac{m_0}{\hbar c}\right) \qquad (6.122)$$

and he remarked that "they have the form of Maxwell's equations […], completed by an external potential (A_r)". But, **in no way** he considered (6.122) as the equations of a photon.

Then, he gives a spin operator, but without calculating its eigenvalues and thus **ignoring that his electron has a spin 1!** Very astonishing because de Broglie worked on one floor above Proca and had deduced, two years before, this value 1 in his equation of describing a massive photon (*Broglie* 6).

§12.2) The Bargmann-Wigner equation.

The Bargmann-Wigner equations for higher values of spin was published in 1948 (see *Bargmann V., Wigner E.P.*) and were identic to the de Broglie's equations published in 1943 (*Broglie 9*). Not identic, indeed because it was **without the idea of fusion and restricted by an *a priori* condition of symmetry,** so as they had only one half of the de Broglie solutions. This may be verified in Lurié's *Particles and Fields* (*Lurié*), where *the equations 1 (97) p. 27* are exactly the equations, taken from de Broglie's *Théorie générale des particules à spin,* p. 138 (*Broglie 9*).

When the general theory is applied to the case of spin 1, Bargmann and Wigner found the equations *1(108a), 1(108b)*, identic to the equation taken from de Broglie's book p. 106 (*Broglie 9*), with a difference : in virtue of their condition of symmetry, Bargmann and Wigner do not develop the wave on the 16 Clifford matrices, as we did in (6,19), but only on 10 of them : γ_μ, $\gamma_{[\mu\nu]}$. The 6 others are forgotten, so that only the spin 1 Maxwell equations (6,21) are obtained, but not the non Maxwellian (6,22), corresponding to the spin 0, which have an important physical meaning as we know.

They would be unable to include the Aharonov-Bohm effect, as we did, and a fortiori to find the magnetic photon of which we proved not only that it has a logical place in the theory, but that it was already hidden into de Broglie's theory and later experimentally observed. And it is the photon that automatically appears in the interaction between the leptonic monopole and the electromagnetic field .

[14] Heisenberg and de Broglie were among the few who immediately adopted Dirac's equation, whatever could be the difficulties with negative energies.